EPS 8th Grade Mathematics Instructional Sequence 2010-11

Unit	Start date:	**8th Grade Core Content Standard**
Thinking with Math Models	Sept. 9	**8.1** *Linear functions and equations.* Solving linear equations and working with linear functions in multiple representations.
Samples and Populations	Oct. 5	**8.3** *Summary and analysis of data sets:* Using knowledge of liner functions, various data plots, variability, and central tendency measures to analyze data
Growing, Growing, Growing	Nov. 18	**8.4** *Additional content*: scientific notation and exponent properties.
Say It With Symbols	Jan. 3	**8.1** *Linear functions and equations*: modeling applied problems with symbolic expressions and equations.
Shapes of Algebra	Feb. 2	**8.1** *Linear functions and equations:* modeling applied problems with symbolic expressions and equations.
Looking For Pythagoras	Feb. 25	**8.2** Properties *of geometric figures:* specifically with triangles and the Pythagorean Theorem, including squares and square roots of numbers.
Kaleidoscopes, Hubcaps, and Mirrors	April 15	**8.2** *Properties of geometric figures:* angle attributes of 2-D figures, geometric transformations in the plane
What Do You Expect	May 26	**8.3** *Summary and analysis of data sets*: determining probabilities, sample spaces, using various counting techniques.

8.5. Core Processes: Reasoning, problem solving, and communication

Students refine their reasoning and problem-solving skills as they move more fully into the symbolic world of algebra and higher-level mathematics. They move easily among representations—numbers, words, pictures, or symbols—to understand and communicate mathematical ideas, to make generalizations, to draw logical conclusions, and to verify the reasonableness of solutions to problems. In grade eight, students solve problems that involve proportional relationships and linear relationships, including applications found in many contexts outside of school. These problems dealing with proportionality continue to be important in many applied contexts, and they lead directly to the study of algebra. Students also begin to deal with informal proofs for theorems that will be proven more formally in high school.

Mathematics content based on
Adopted Washington State K-8 Mathematics Standards, April 28, 2008, Office of the Superintendent of Public Instruction

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Grade 8

Adopted Washington State

Mathematics Standards

April 28, 2008

8.1. Core Content: Linear functions and equations (Algebra)

Students solve a variety of linear equations and inequalities. They build on their familiarity with proportional relationships and simple linear equations to work with a broader set of linear relationships, and they learn what functions are. They model applied problems with mathematical functions represented by graphs and other algebraic techniques. This Core Content area includes topics typically addressed in a high school algebra or a first-year integrated math course, but here this content is expected of all middle school students in preparation for a rich high school mathematics program that goes well beyond these basic algebraic ideas.

8.2. Core Content: Properties of geometric figures (Numbers, Geometry/Measurement)

Students work with lines and angles, especially as they solve problems involving triangles. They use known relationships involving sides and angles of triangles to find unknown measures, connecting geometry and measurement in practical ways that will be useful well after high school. Since squares of numbers arise when using the Pythagorean Theorem, students work with squares and square roots, especially in problems with two- and three-dimensional figures. Using basic geometric theorems such as the Pythagorean Theorem, students get a preview of how geometric theorems are developed and applied in more formal settings, which they will further study in high school.

8.3. Core Content: Summary and analysis of data sets (Algebra, Data/Statistics/Probability)

Students build on their extensive experience organizing and interpreting data and begin to apply statistical principles to analyze statistical studies or short statistical statements, such as those they might encounter in the newspaper, on television, or on the Internet. They use mean, median, and mode to summarize and describe information, even when these measures may not be whole numbers. Students use their knowledge of linear functions to analyze trends in displays of data. They create displays for two sets of data in order to compare the two sets and draw conclusions. They expand their work with probability to deal with more complex situations than they have previously seen. These concepts of statistics and probability are important not only in students' lives, but also throughout the high school mathematics program.

8.4. Additional Key Content (Numbers, Operations)

Students deal with a few key topics about numbers as they prepare to shift to higher level mathematics in high school. First, they use scientific notation to represent very large and very small numbers, especially as these numbers are used in technological fields and in everyday tools like calculators or personal computers. Scientific notation has become especially important as "extreme units" continue to be identified to represent increasingly tiny or immense measures arising in technological fields. A second important numerical skill involves using exponents in expressions containing both numbers and variables. Developing this skill extends students' work with order of operations to include more complicated expressions they might encounter in high school mathematics. Finally, to help students understand the full breadth of the real-number system, students are introduced to simple irrational numbers, thus preparing them to study higher level mathematics in which properties and procedures are generalized for the entire set of real numbers.

Grade Eight Performance Expectations

Linear Functions and Equations (algebra)

8.1.A **Solve one-variable linear equations.** (7.1.E)

8.1.B **Solve one- and two-step linear inequalities and graph the solutions on the number line.**

8.1.C **Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.**

8.1.D **Determine the slope and y-intercept of a linear function described by a symbolic expression, table, or graph.**

8.1.E **Interpret the slope and y-intercept of the graph of a linear function representing a contextual situation.**

8.1.F **Solve single- and multi-step word problems involving linear functions and verify the solutions.**

8.1.G *Determine and justify whether a given verbal description, table, graph, or symbolic expression represents a linear relationship.*

> The performance expectation identified in the parentheses represents a connection to a previous or future grade level performance expectation.
>
> **Bold** and *italicized* formatting based on the most current version of the MSP Mathematics Item Specifications. Expectations for the state assessment are in **bold text**. Expectations for local instruction and assessment appear in *italicized* text.
> ** This performance expectation may be included in items assessing process performance expectaions.

Properties of Geometric Figures (numbers, geometry/measurement)

8.2.A **Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.**

8.2.B **Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.**

8.2.C *Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and* **apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.**

8.2.D **Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.**

8.2.E *Quickly* **recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers.** **

8.2.F *Demonstrate the Pythagorean Theorem and its converse and* **apply them to solve problems.**

8.2.G **Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.**

Summary and Analysis of Data Sets (algebra, data/statistics/probability)

8.3.A **Summarize and compare data sets in terms of variability and measures of center.** (7.4.C)

8.3.B **Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.** (7.4.D)

8.3.C **Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.**

8.3.D *Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.* (7.4.E)

8.3.E *Determine whether conclusions of statistical studies reported in the media are reasonable.* (7.4.E)

8.3.F **Determine probabilities for mutually exclusive, dependent, and independent events from small sample spaces.** (7.4.B)

8.3.G **Solve single- and multi-step problems using counting techniques and Venn diagrams and verify the solutions.**

Additional Key Content (numbers, operations)

8.4.A **Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.**

8.4.B **Solve problems involving operations with numbers in scientific notation and verify solutions.**

8.4.C **Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.** (6.2.D)

8.4.D *Identify rational and irrational numbers.*

Reasoning, Problem Solving, and Communication

8.5.A *Analyze a problem situation to determine the question(s) to be answered.* (7.6.A)

8.5.B **Identify relevant, missing, and extraneous information related to the solution to a problem.** (7.6.B)

8.5.C *Analyze and compare mathematical strategies for solving problems, and* **select and use one or more strategies to solve a problem.** (7.6.C)

8.5.D **Represent a problem situation,** *describe the process used to solve the problem, and* **verify the reasonableness of the solution.** (7.6.D)

8.5.E *Communicate the answer(s) to the question(s) in a problem using appropriate representations, including symbols and informal and formal mathematical language.* (7.6.E)

8.5.F *Apply a previously used problem-solving strategy in a new context.* (7.6.F)

8.5.G **Extract and organize mathematical information from symbols, diagrams, and graphs to make inferences, draw conclusions, and justify reasoning.** (7.6.G)

8.5.H *Make and test conjectures based on data (or information) collected from explorations and experiments.* (7.6.H)

Thinking with Mathematical Models

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations
1.1 Testing Paper, p.5	1		Optional	**8.1.C Represent a linear function with a verbal description, table, graph, or symbolic expression, and make connections among these representations.** **8.1.D Determine the slope and y-intercept of a linear function described by a symbolic expression, table, or graph.** **8.1.E Interpret the slope and y-intercept of the graph of a linear function representing a contextual situation.** **8.1.F Solve single- and multi-step word problems involving linear functions and verify the solutions.** *8.1.G Determine and justify whether a given verbal description, table, graph, or symbolic expression represents a linear relationship.* **8.3.C Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.** (Introduction of what a scatter plot is in this book which is taught with lines of best fit. Creating scatter plot will be taught in Samples and Populations.) Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
1.2 Drawing Graph Models, p.7	1		Must do	
1.3 Finding Equation Models, p9	1		Should do	
1.4 Setting the Right Price, p.10	1		Optional	
1.5 Writing Equations for Lines, p.12	2		Must do	
Practice with slope, intercept, & equations: **CMP2 TWMM,** • Additional Practice, Inv. 2, p. 10 – 15 • Skill: Using Linear Models, Inv. 2, p. 16- 17 • Skill: Writing Equations of Lines, Inv. 2, p. 18 – 19 **CMP2 MSA** • Skill: Linear Relationships, Inv. 1, p. 86 • Additional Practice, Inv. 4, p. 102, 104 • Skill: Writing Equations, p. 113	2	Binder and CMP2 Disk		
Inv. 1 Reflections, p.25	1			
Check-up	1			
2.1 Testing Bridge Lengths, p.26	1		Optional	
4.1 Modeling Real-life Events, p.47	1		Optional	
4.2 Writing Stories to Match Graphs, p.49	1		Optional	
4.3 Exploring Graphs, p. 51	1		Optional	
Inv. 4 Reflections, p.59	1			
Review for Unit Assessment	1			
Unit Assessment	1			
Total Instructional Days	17 Days			

Contents in Thinking with Mathematical Models

Name ______________________________ Date ____________ Class ____________

Additional Practice

Investigation 2

Thinking With Mathematical Models

For Exercises 1–4, write an equation and sketch a graph for the line that meets the given conditions.

1. A line with slope 3.5 and y-intercept $(0, 4)$

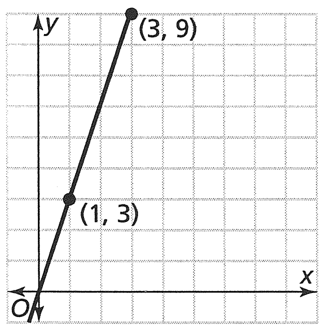
2. A line with slope $\frac{3}{2}$ that passes through the point $(-2, 0)$

3. A line that passes through the points $(2, 7)$ and $(6, 15)$

4. A line that passes through the points $(2, 1)$ and $(6, 9)$

For Exercises 5–8, write an equation for the line shown. Identify the slope and *y*-intercept.

5.



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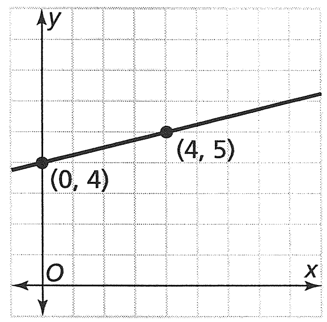

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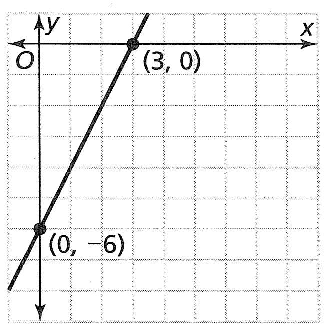

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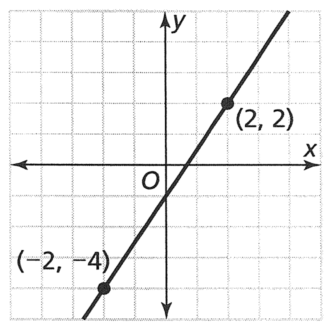


9. For parts (a)–(c), write an equation and sketch a graph for the line that meets the given conditions. Use one set of axes for all three graphs.

a. A line with slope $\frac{2}{3}$ and y-intercept $(0, 0)$

b. A line with slope $\frac{2}{3}$ that passes through the point $(6, 6)$

c. A line with slope $\frac{2}{3}$ that passes through the point $(6, 2)$

d. What do you notice about the equations and graphs of the three lines?

Additional Practice *(continued)*

10. For parts (a)–(c), write an equation and sketch a graph for a line that meets the given conditions. Use one set of axes for all three graphs.

a. A line with slope 3 and y-intercept $(0, 5)$

b. A line parallel to the line drawn in part (a) with a y-intercept greater than 5

c. A line parallel to the line drawn in parts (a) and (b) with a y-intercept less than 5

d. What do you notice about the equations and graphs of the three lines?

For Exercises 11–12, write an equation and sketch a graph for the line that meets the given conditions.

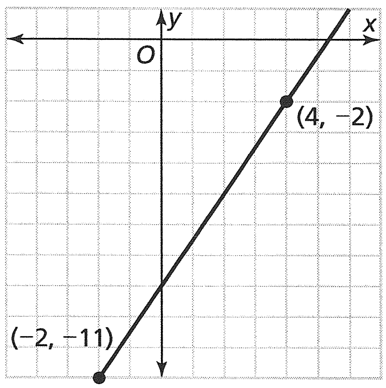
11. A line with slope $-\frac{15}{5}$ that passes through the point $(-2.5, 4.5)$

12. A line that passes through the points $(2, -9)$ and $(-2, 3)$

Additional Practice *(continued)*

For Exercises 13–14, write an equation for the line shown. Identify the slope and *y*-intercept.

13.



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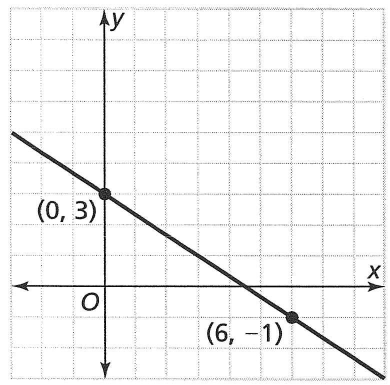


15. For parts (a)–(c), write an equation and sketch a graph for the line that meets the given conditions. Use one set of axes for all three graphs.

a. A line with slope -2 and y-intercept $(0, 0)$

b. A line with slope -2 that passes through the point $(3, -3)$

c. A line with slope -2 that passes through the point $(3, -9)$

d. What do you notice about the equations and graphs of the three lines?

Additional Practice *(continued)*

16. For parts (a)–(c), write an equation and sketch a graph for a line that meets the given conditions. Use one set of axes for all three graphs.

a. A line with slope $-\frac{1}{2}$ and y-intercept $(0, 3)$

b. A line parallel to the line drawn in part (a) with a y-intercept greater than 3

c. A line parallel to the line drawn in parts (a) and (b) with a y-intercept less than 3

d. What do you notice about the equations and graphs of the three lines?

17. a. Predict how high a stack of 10 cups would be.

Stack of Styrofoam Cups

Number of Cups	1	2	3	4
Height of the Stack of Cups (cm)	7	8	9	10

b. Describe the pattern in words.

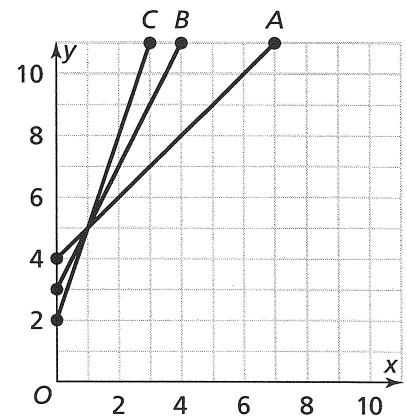
c. Describe the pattern with an equation. Let x represent the number of cups and h the height.

d. What does the coefficient of x mean in this context? Does it have a unit of measure? Explain.

e. What does the constant term mean in this context? Does it have a unit of measure? Explain.

Additional Practice *(continued)*

18. To the right are the graphs of three lines.



a. Match each line with its rule.

$y = x + 4$ $\quad$ $y = 2x + 3$ $\quad$ $y = 3x + 2$

b. For each equation, what are the y-values when $x = 3$? When $x = 4$?

c. Why are the y-values "farther apart" when $x = 4$ than when $x = 3$?

19. Find exact solutions for each of these equations.

a. $9 - x = 3x - 7$ $\quad$ **b.** $3.6x + 2.4 = 2.1x - 0.6$

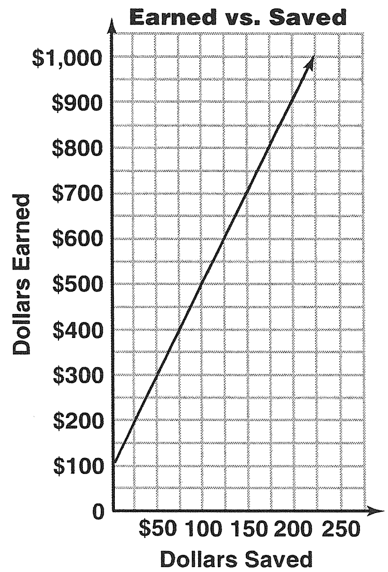
20. Find at least three values of x for which the inequality is true.

a. $5x - 3 \leq 12$ $\quad$ **b.** $8x - 1 \leq 4x + 7$

Name ______________________________ Date __________ Class __________

Skill: Using Linear Models

For Exercises 1–5, use the graph at the right.



1. What earnings will produce \$225 in savings?

2. How much is saved from earnings of \$400?

3. What is the slope of the line in the graph?

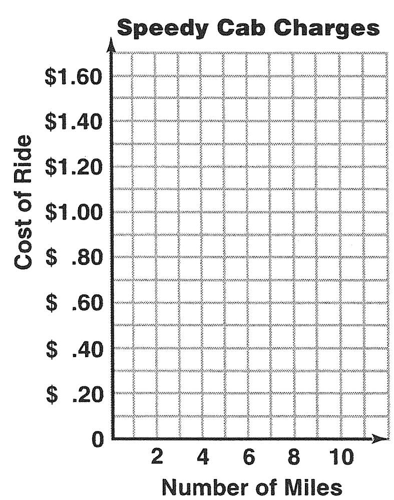
4. For each increase of \$200 in earnings, what is the increase in savings?

5. Write an equation for the line.

6. A ride in a cab costs \$0.40 plus \$0.15 per mile.
   a. Write and graph an equation for traveling x miles in the cab.

   b. The cab charges \$0.70 for a ride of how many miles?

   c. How much does the cab charge for a trip of 8 miles?



Skill: Using Linear Models *(continued)*

A giraffe was 1 foot tall at birth, 7 feet tall at the age of 4, and $11\frac{1}{2}$ feet tall at the age of 7.

7. Plot the data.

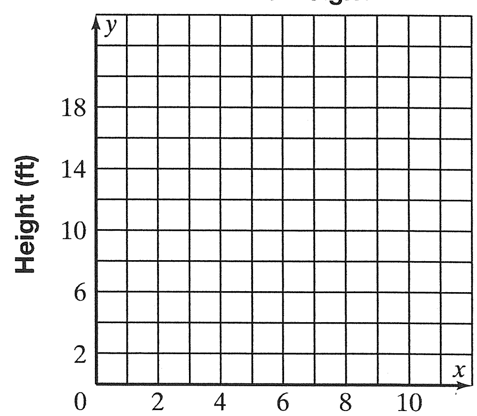
8. Draw a line that models the pattern in the data.

9. Write an equation for your line.

10. Use your equation to find the following information.
    a. the giraffe's height at the age of 5

    b. the age at which the giraffe was 16 ft tall

Giraffe Height



A hippopotamus weighed 700 pounds at the age of 1, 1,900 pounds at the age of 3, and 2,500 pounds at the age of 4.

11. Plot the data.

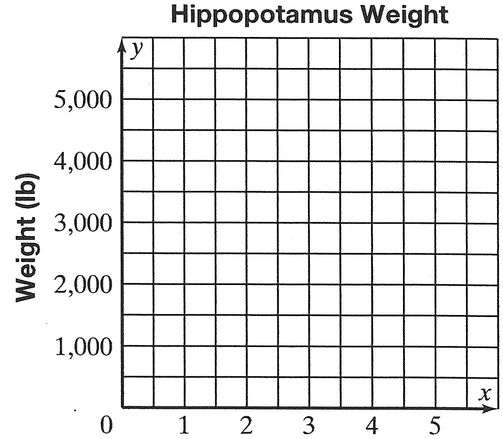
12. Draw a line that models the pattern in the data.

13. Write an equation for your line.

14. Use the equation to predict the following information.
    a. the hippo's weight at the age of 8

    b. the age at which the hippo weighed 7,900 pounds

Hippopotamus Weight



Name ______________________________ Date __________ Class __________

Skill: Writing Equations of Lines

Write an equation for the line through the given points or through the given point with the given slope.

1. $(5, 7), (6, 8)$

2. $(-2, 3)$; slope $= -1$

3. $(1, 2), (3, 8)$

4. $(-2, 3)$; slope $= 4$

5. $(4, 7)$; slope $= \frac{3}{2}$

6. $(6, -2)$; slope $= -\frac{4}{3}$

7. $(0, 5), (-3, 2)$

8. $(8, 11), (6, 16)$

Name ______________________________ Date __________ Class __________

Skill: Writing Equations of Lines *(continued)*

Investigation 2

Thinking With Mathematical Models

Is the relationship shown by the data linear? If it is, model the data with an equation.

9.

x	y
2	3
3	7
4	11
5	15

10.

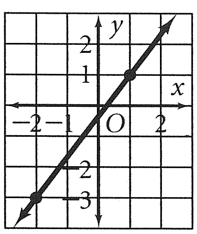
x	y
−3	4
−1	6
1	7
3	10

11.

x	y
−2	5
3	−5
7	−13
11	−21

Write an equation of each line.

12.



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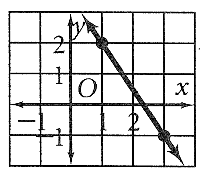

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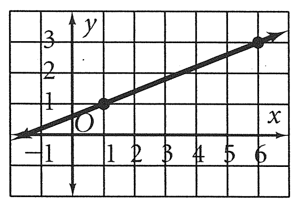


Name ______________________________ Date __________ Class __________

Skill: Linear Relationships

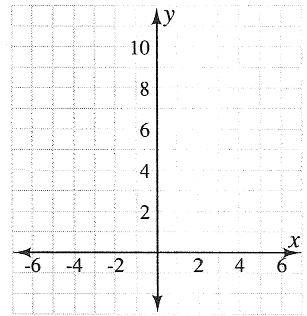
Investigation 1

Moving Straight Ahead

Does the point represent a point on the graph of $y = x - 4$.

1. $(0, -4)$ **2.** $(5, -1)$ **3.** $(-3, -7)$ **4.** $(-7, -3)$

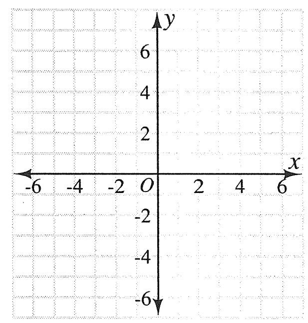
5. You order books through a catalog. Each book costs $12 and the shipping and handling cost is $5. Write an equation and make a graph that represents your total cost.



a. What is the total cost if you buy 6 books?

b. What is the total cost if you buy 4 books?

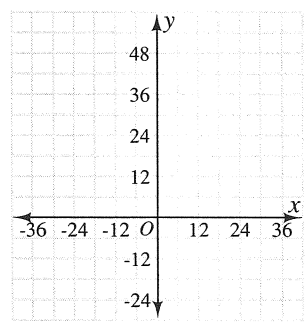
6. A ride in a taxicab costs $2.50 for the first mile and $1.50 for each additional mile, or part of a mile. Write an equation and make a graph that represents the total cost.



a. What is the total cost of a 10-mile ride?

b. What is the total cost of a 25-mile ride?

7. A tree is 3 feet tall and grows 3 inches each day. Write an equation and make a graph that represents how much the tree grows over time.

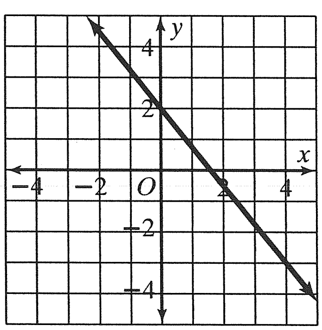


a. How tall is the tree in a week?

b. How tall is the tree in 4 weeks?

Write an equation for each graph.

8.



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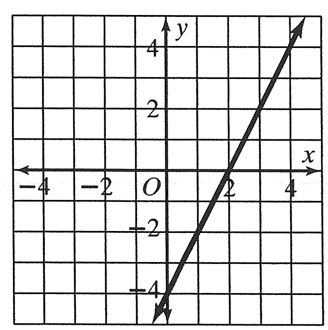


Name ______________________ Date ____________ Class ____________

Additional Practice

Investigation 4

Moving Straight Ahead

1. Find the slope and y-intercept of the line represented by each equation.

a. $y = 2x - 10$ **b.** $y = 4x + 3$ **c.** $y = 4x - 4.5$

d. $y = 2.6x$ **e.** $y = 7x + 1$

2. Each table in (i.)–(v.) below represents a linear relationship. Do parts (a)–(c) for each table.

a. Find the slope of the line that represents the relationship.

b. Find the y-intercept for the graph of the relationship.

c. Determine which of the following equations represents the relationship:

$y = 3 - 4x$ $y = x + 6$ $y = 4x - 3$ $y = 3x - 1.5$ $y = 2.5x$

i.

x	y
0	0
1	2.5
2	5
3	7.5
4	10

ii.

x	y
0	6
1	7
2	8
3	9
4	10

iii.

x	y
0	$^-1.5$
1	1.5
2	4.5
3	7.5
4	10.5

iv.

x	y
0	3
1	$^-1$
2	$^-5$
3	$^-9$
4	$^-13$

v.

x	y
1	1
2	5
3	9
4	13
5	17

Additional Practice *(continued)*

5. On Saturdays, Jim likes to go to the mall to play video games or pinball. Round-trip bus fare to and from the mall is \$1.80. Jim spends \$0.50 for each video or pinball game.

a. Write an equation for the amount of money M it costs Jim to go to the mall and play n video or pinball games.

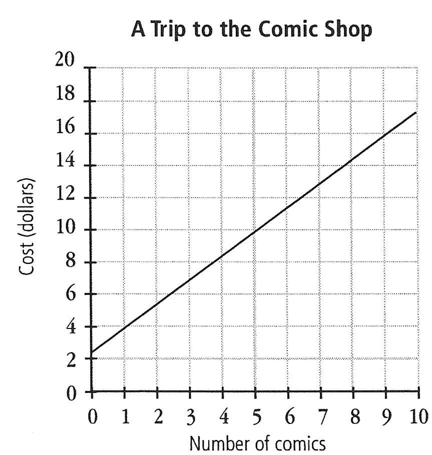
b. What is the slope of the line your equation represents? What does the slope tell you about this situation?

c. What is the y-intercept of the line? What does the y-intercept tell you about the situation?

d. How much will it cost Jim to travel to the mall and play 8 video or pinball games?

e. If Jim has \$6.75, how many video or pinball games can he play at the mall?

6. At the right is a graph showing the total cost (including bus fare and the cost of comics) for Angie to go to the Comic Shop to buy new comic books.



a. What is Angie's round-trip bus fare? Explain your reasoning.

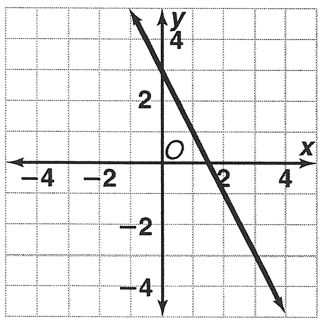
b. How much does a comic book cost at the Comic Shop? Explain.

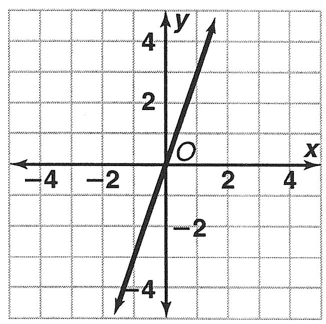
c. Write an equation that shows how much money M it costs Angie to buy n comic books at the Comic Shop. What information did you use from the graph to write the equation?

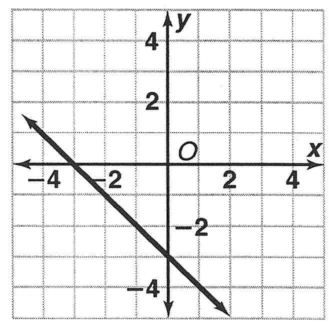
Name ______________________________ Date ____________ Class ____________

Skill: Writing Equations

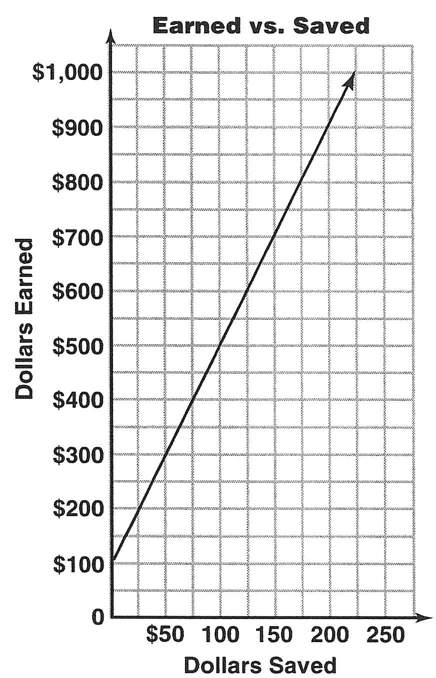
Write an equation for each line.

1. 

2. 

3. 

Use the graph at the right for Exercises 4–8.



4. What earnings will produce $225 in savings?

5. How much is saved from earnings of $400?

6. What is the slope of the line in the graph?

7. For each increase of $200 in earnings, what is the increase in savings?

8. Write an equation for the line.

Samples & Populations

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations
Review stem & leaf, histograms, circle graphs, bar graphs, line plots, measures of center, and variability. **The first 3 pages of this section is a brief summary of each of these types of graphs.** Can choose from: • Data Distributions CMP2 Additional Practice Inv. 1-3; • Data About Us CMP2 Additional Practice Inv. 2; • Bridge to Algebra lesson 11.5 • Navigations through Data Analysis: Students and Basketball Players • Navigations through Data Analysis: Batteries	4	Binder or CMP2 disk All of these are included in the binder		**8.3.A Summarize and compare data sets in terms of variability and measures of center.** **8.3.B Select, construct, and analyze data displays, including box-and-whisker plots, to compare two sets of data.** **8.3.C Create a scatterplot for a two-variable data set, and, when appropriate, sketch and use a trend line to make predictions.** *8.3.D Describe different methods of selecting statistical samples and analyze the strengths and weaknesses of each method.* *8.3.E Determine whether conclusions of statistical studies reported in the media are reasonable.* Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
1.2 Using Box-and-Whisker Plots, p. 7 (important vocab.)	2		Should do	
1.3 Comparing Prices, p. 10	1		Must do	
1.4 Making a Quality Choice, p. 12	1		Must do	
1.5 Comparing Quality & Price, p. 12-13	1		Optional	
Inv. 1 Reflections, p. 23	1			
Check-Up	1			
(CMP2) Samples and Populations Problem 4.2	1	Binder or CMP2 disk		
(CMP2) Samples and Populations Problem 4.3	1	Binder or CMP2 disk		
ACE CMP1 pgs. 20—22 #8, 9 and Navigations through Data Analysis: Reading a Scatter Plot	1	Binder		
Comparing Graphs- Navigations through Data Analysis: Migraines Histograms	1	Binder		
Navigations through Data Analysis: Migraines Box Plots	1	Binder		
ACE Questions CMP1; pgs. 32 - 33 #10, 11, pgs. 56-57 #3, 4 **(CMP2)**: pg. 40 #22, 23, 24, 25	1	Binder or CMP2 disk		
Quiz	1			
2.1 Asking About Honesty, p. 25	1		Should do	
2.2 Selecting a Sample, p. 26	1		Must do	
2.3 Asking the Right Questions, p. 29	1		Must do	
Inv. 2 Reflections, p. 36	1			
3.1 Choosing Randomly, p. 37	1		Must do	

3.2 Selecting a Random Sample, p. 38	1		Optional	
3.3 Choosing a Sample Size, p. 41	1		Optional	
Review for Unit Assessment	1			
Unit Assessment	1			
Total Instructional Days for Samples & Populations:	27			

Contents in Samples & Populations

- Three page summary of Types of Graphs
- Additional Practice, Inv 1 p. 156-161
- Additional Practice, Inv 2 p. 164
- Additional Practice, Inv 3 p. 167-170
- Additional Practice, Inv 2 p. 117-119
- Bridge to Algebra lesson 11.5 p. 363A-B,363-365, 365 A-B
- Navigations: Students & Basketball Players teacher p. 37-38; student p. 88
- Navigations: Batteries teacher p. 39-42; student p. 89
- CMP2 Lesson 4.2 p. 63—65, p. 99, 101-102
- CMP2 Lesson 4.3 p. 66-68, p. 103, 105-106
- Navigations: Reading a Scatterplot teacher p. 71-72, 2 student pages
- Navigations: Migraine- Histograms teacher p. 60-62, student pages 94-96
- Navigations: Migraine- Box Plots teacher p. 63-65, student p. 97-99
- CMP2 Ace p. 40 #22-25

Fig. **1.2.**
Types of graphs

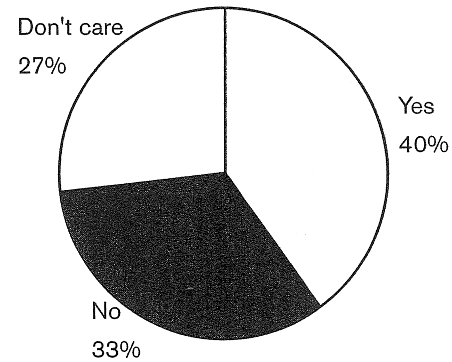
Type of Graph	Example

A *circle graph,* or *pie chart,* is a circle divided into parts, or sectors or wedges. Each part shows the percent of the data elements that are categorized similarly (e.g., grouped into intervals). The parts must sum to 100 percent. Circle graphs are often difficult to make, since each percent must be converted to an angle (i.e., the appropriate fraction of 360°) and the angles are sometimes difficult to draw.

(Moore 1991, pp. 180–81)

Circle Graph

Responses to Julio's Question



A *stem plot* (also called a *stem-and-leaf plot*) is a display that is most often used to "separate" the tens digits from the ones digits of the data values. The tens digits are called the *stems,* and the ones digits are called the *leaves*. Each leaf represents one of the data elements. Ordering the leaves on each stem from least to greatest often facilitates the interpretation of this display. This display works best when the data set contains more than twenty-five elements and when the data values span several decades of values. A stem plot can also be adapted to show simple decimal values—for example, whole numbers and tenths. A back-to-back stem plot can be used to compare two data sets.

(Landwehr 1986, pp. 7–9, 33)

Stem Plot

Battery Life (in Hours) for Two Brands

Always Ready Batteries		Tough Cell Batteries
9	1	
2	2	
1	3	
4	4	
2	5	
3	6	
4	7	
9, 6, 6, 4	8	4, 6, 8
9, 6, 6, 5, 3, 2	9	1, 1, 4, 5, 5, 6, 7, 7, 8, 9
7, 6, 5, 5, 1, 1	10	0, 1, 2, 3, 3, 3, 4, 5, 6, 7, 7, 8, 9
7, 6, 5, 5, 5, 5, 5, 4, 3, 1, 1, 0, 0, 0, 0	11	0, 0, 0, 0, 1, 2, 4, 5, 5
	12	1, 1, 1
	13	0
	14	0
	15	
	16	
5	17	
	18	
	19	
3	20	

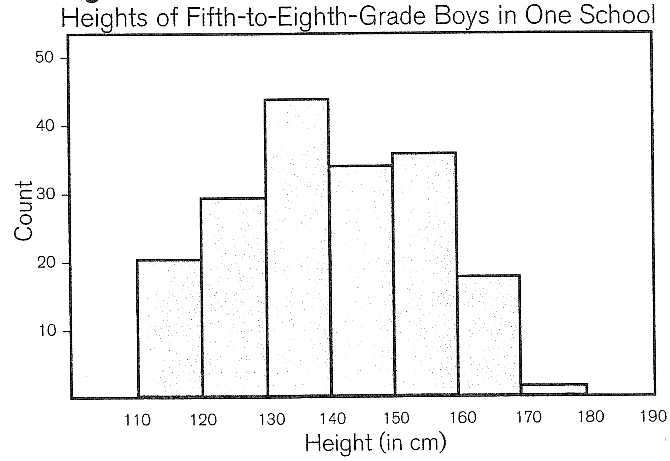
On the left side, 6 | 8 means 86. On the right side, 8 | 6 means 86.

A *histogram* is used when data elements could assume any value in a range—heights or weights of people, for example. The data are organized in equal intervals; the data values are marked on the horizontal axis. Bars of equal width are drawn for each interval, with the height of each bar representing either the number of elements or the percent of elements in that interval; the number or percent is marked on the vertical axis. The bars are drawn without any space between them.

(Moore 1991, pp. 191–92)

Histogram

Heights of Fifth-to-Eighth-Grade Boys in One School



Continued

Fig. **1.2.**

Types of graphs

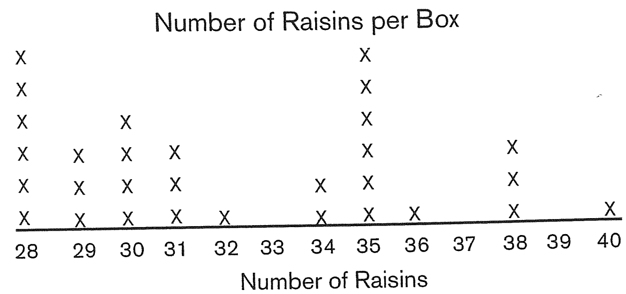
Different graphs are used in different situations; each has both advantages and disadvantages. For example, some graphs are useful for small data sets, whereas others are useful for large data sets. Some graphs display each data value individually, but others "hide" individual values in bars or other visual elements. This chart contains important information about the graphs that middle-grades students are likely to use.

Type of Graph	Example

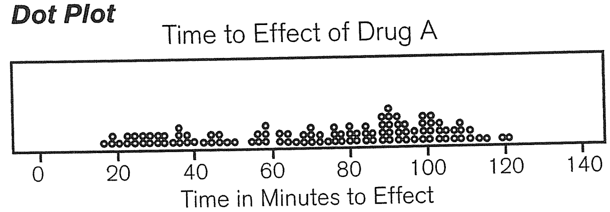
A *line plot* is a fast way to organize data. The possible data values are listed on a horizontal axis, and one *X* for each element in the data set is placed above the corresponding value. This display works best when the data set has fewer than twenty-five elements and when the range of possible values is not too great. A *dot plot* is similar to a line plot; small dots are used instead of *X*s.

(Landwehr 1986, p. 5)

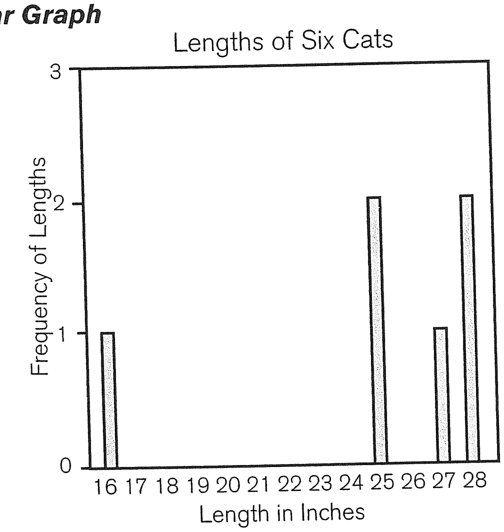
Line Plot



Dot Plot



Bar Graph



A *bar graph* shows the frequencies of specific data values in a data set. It can be used for categorical or numerical data, but it is one of the most common ways to display categorical data. The length of the bar drawn for each data value represents the frequency of that value. Bars may be drawn vertically or horizontally. To avoid confusion, the bars should be the same width. In elementary school mathematics, a *case-value plot* is sometimes created. In a case-value plot, the height of the bar drawn for each data element represents the data value. Bar graphs and case-value plots are not interpreted in the same ways, and sometimes students confuse the interpretation of these two displays.

(Moore 1991, pp. 184–85)

Case-Value Plot

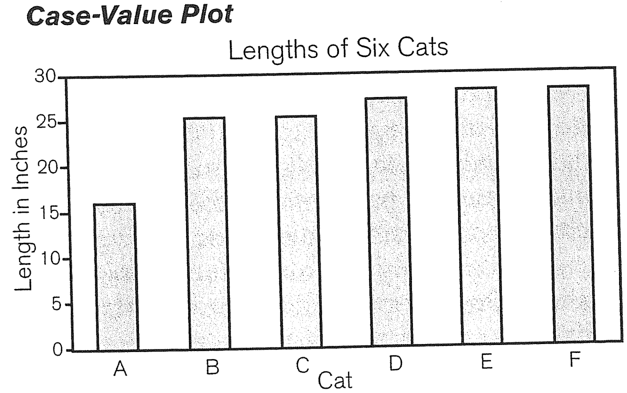


Fig. **1.2.**

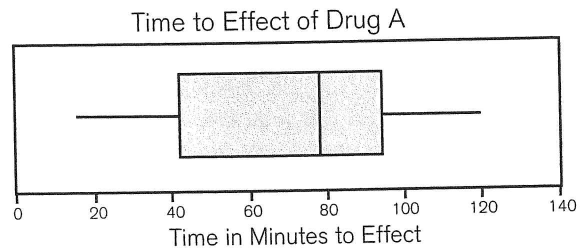
Types of graphs

Type of Graph	Example

A *box plot* (also called a *box-and-whiskers plot*) is constructed by marking the "five-point summary" (i.e., the least and greatest values, the median, and the first and third quartiles), drawing a box to capture the interval from the first to the third quartile, and connecting the box to the least and greatest values with line segments. The data elements are not displayed individually, which makes it impossible to determine if there are gaps or clusters in the data. Box plots are very useful, however, for comparing data sets, especially when the data sets are large or when they have different numbers of data elements.

(Landwehr 1986, pp. 57, 73)

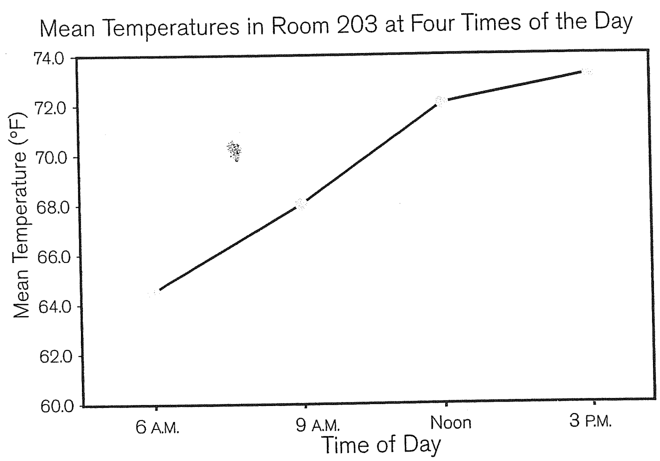
Box Plot



A *line graph* is typically used for continuous data to show the change in a variable—over time, for example. The time is marked on the horizontal axis, and the values of the variable are marked on the vertical axis. Each element of the sample is associated with a value for time and a value of the variable. Each pair of values is graphed, and the points are connected with line segments. It is important to look carefully at the scale marked on the vertical axis, since changing the scale of the vertical axis can dramatically change the visual impression of the graph.

(Moore 1991, pp. 181–83)

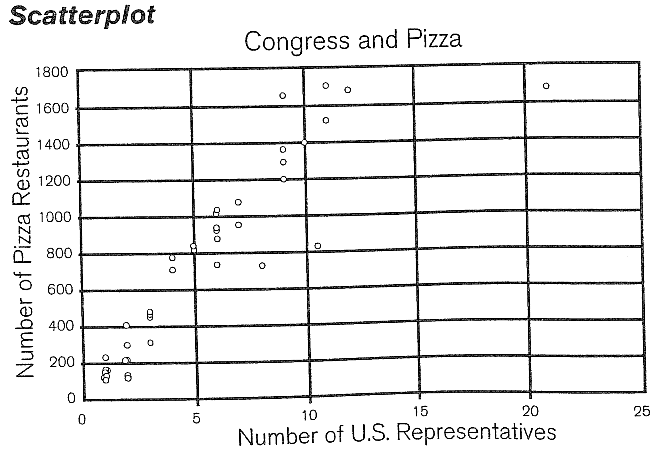
Line Graph



A *scatterplot* is used when two measurements are made for each element of the sample. The graph consists of points on a two-dimensional grid; the two coordinates of each point are determined by the two measurements for the corresponding element of the sample. A scatterplot is one of the best ways to determine if two characteristics are related.

(Landwehr 1986, pp. 84–86, 137)

Scatterplot



Name ______________________________ Date ____________ Class ____________

Additional Practice

Investigation 1

Data Distributions

For Exercises 1 and 2, use the table below.

New M&M's® Candies

Bag #	Green	Yellow	Orange	Blue	Brown	Red	Total
1	14	7	8	10	17	3	59
2	14	10	16	7	7	3	57
3	14	17	7	11	9	2	60
4	13	13	8	11	6	7	58
5	15	11	7	15	6	5	59
6	11	6	16	14	5	5	57
7	20	9	8	13	7	2	59
8	10	14	8	14	10	3	59
9	17	11	8	14	10	3	63
10	17	10	14	14	4	2	61
11	14	11	11	5	9	7	57
12	9	7	20	8	12	1	57
13	12	13	9	17	7	2	60
14	8	8	12	11	17	4	60
15	18	8	13	9	7	4	59
TOTAL							

1. a. On a separate piece of paper, make a bar graph for each set of data for Bags 1, 2, and 3. Each bar graph shows the percent of all candies for each color found in that bag.

b. Write two or more comparison statements that describe the data for the three bags of candy.

2. a. Determine the totals for each color of M&M's candies found in all 15 bags. On a separate piece of paper, make a bar graph for these data that shows percent of all candies for each color found in the fifteen bags.

b. Describe the data by writing two or more comparison statements.

c. Compare this graph with the graphs you made for the Bags 1, 2, and 3 of M&M's candies. Is there some plan to the distribution of colors in bags of M&M's candies? Explain your reasoning.

For Exercises 3–5, use the table of data.

Immigrants to the United States

Decade	Immigrants From Canada	Total Immigrants	Percent of Immigrants From Canada
1820	209	8,385	2%
1821–30	2,277	143,439	2%
1831–40	13,624	599,125	2%
1841–50	41,723	1,713,251	2%
1851–60	59,309	2,598,214	2%
1861–70	153,878	2,314,824	7%
1871–80	383,640	2,812,191	14%
1881–90	393,304	5,246,613	7%
1891–1900	3,311	3,687,564	0%
1901–10	179,226	8,795,386	2%
1911–20	742,185	5,735,811	13%
1921–30	924,515	4,107,209	23%
1931–40	108,527	528,431	21%
1941–50	171,718	1,035,039	17%
1951–60	377,952	2,515,479	15%
1961–70	413,310	3,321,677	12%
1971–80	169,939	4,493,314	4%
1981–90	156,938	7,338,062	2%
1991–1996	127,481	9,095,417	1%

Source: Brownstone, D. M. & Franck, I. M. (2001). *Facts about the American immigration,* Bronx, NY: H. W. Wilson, p. 487.

3. a. In each of the decades from 1911–1920 and 1941–1950, how many people immigrated from Canada?

b. Add these bars to the bar graph of Exercise 5 titled, "Number of Immigrants from Canada."

Additional Practice *(continued)*

c. Which of the statements below are true?

- **i.** There are more immigrants who came to the U.S. in the decade between 1911–20 than in 1941–50.
- **ii.** About the same number of immigrants came to the U.S. in the decade between 1911–20 and in the decade between 1941–50.
- **iii.** The number of immigrants in the decade between 1911-20 is about 250,000 more than the number of immigrants who came to the U.S. in the decade between 1941-50.
- **iv.** None of the above is true.
- **v.** All of the above are true.

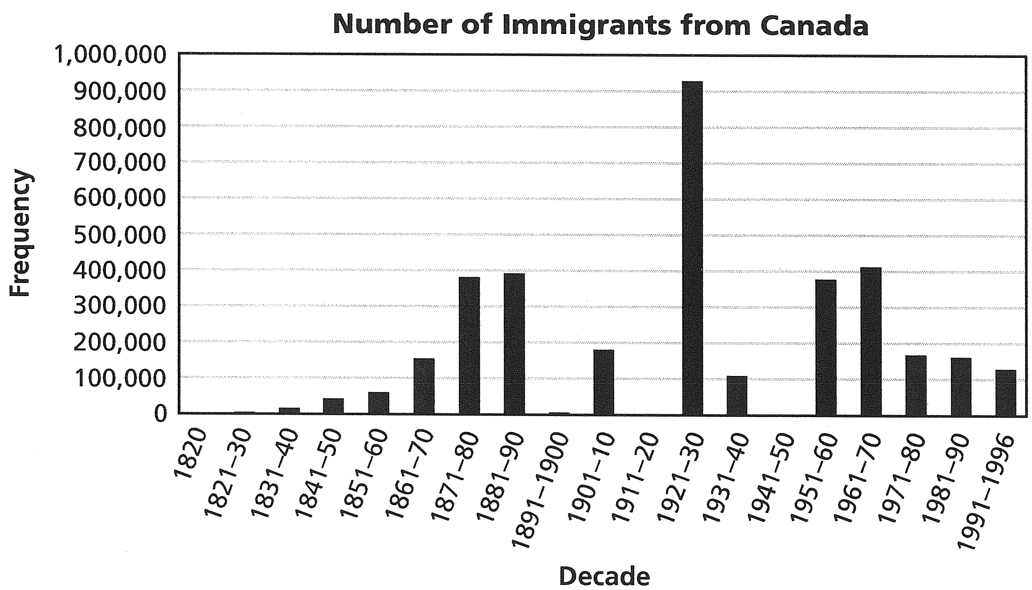
4. a. In each of the decades from 1911-1920 and 1941-1950, how many people were immigrants to the U.S. from all countries?

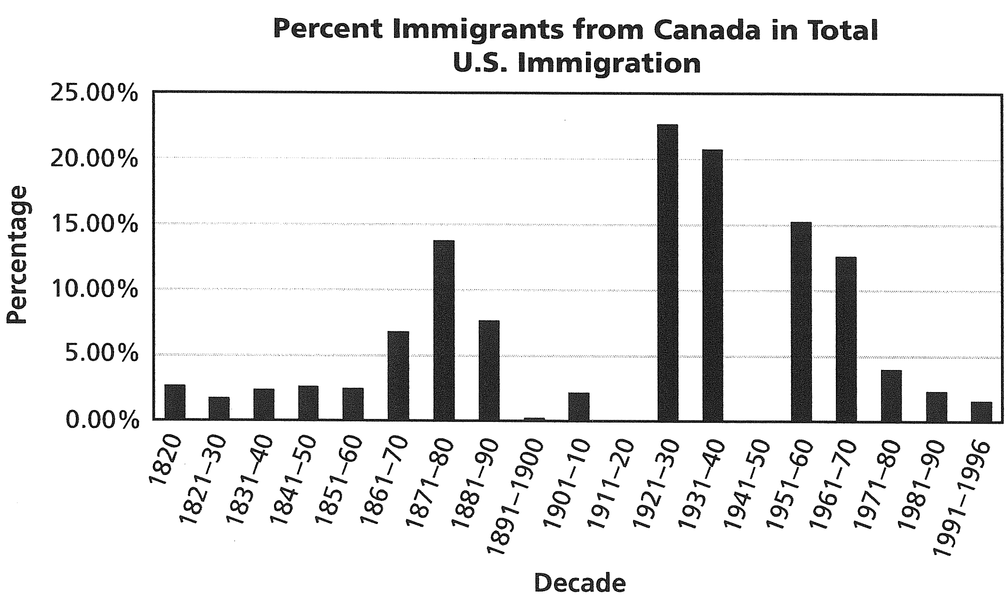
b. What percent of each of these numbers were immigrants from Canada?

c. Add these bars to a copy of the bar graph from Exercise 5 titled, "Percentage of Immigrants from Canada in Total U.S. Immigration."

d. Write two comparison statements about how these data values are similar to or different from the data values for other decades.

5. a. How has the pattern of immigration from Canada to the United States changed between 1820 and 1996? Explain.





b. What does it mean that the bar for 1931–40 looks so different on the two bar graphs above?

For Exercise 6, use the table.

US Population by Region (in millions)

	Northeast	Midwest	South	West	TOTAL
1980	49.1	58.9	75.4	43.2	
1985	49.9	58.8	81.4	47.8	
1990	50.8	59.7	85.5	52.8	
1995	51.5	61.7	92.0	57.7	

Percent of US Population by Region

	Northeast	Midwest	South	West
1980				
1985				
1990				
1995				

6. a. For each year, determine the total population and percent of the total population found in each region. Record this information in the two tables.

   b. On a separate paper, make a bar graph for each region showing the percent of population for each of the four years shown. You will have four bar graphs, each of which has four bars, one for each of the years 1980, 1985, 1990, and 1995.

   c. Which region had the greatest increase in numbers of people in the population from 1980 to 1995? Which region had the smallest increase in numbers of people in the population from 1980 to 1995?

   d. Which region had the greatest increase in percentage of total population from 1980 to 1995? Which region had the greatest decline in percentage of total population from 1980 to 1995?

   e. Write two or more comparison statements that describe the data for the four years.

   f. Which statements below are true?

      i. The South had the most people in each year.

      ii. The population in the Northeast increased from 1980 to 1995.

      iii. The percentage of population in the Northeast increased from 1980 to 1995.

      iv. The distribution of population is more uneven in 1995 than in 1980.

7. Make a line plot, matching the criteria below, to show the distribution of hand widths in a class:

There are 20 students in the class.

The range of hand widths is from 8 cm to 12.5 cm.

The mode hand width is 9.5 cm; there are 6 values at the mode.

The median hand width is 9 cm.

8. Make a line plot, matching the criteria below, to show the distribution of hand widths in a class:

There are 20 students in the class

The range of hand widths is from 8 cm to 12.5 cm.

The mode hand width is 9.5 cm; there are 6 values at the mode.

The median hand width is 10 cm.

Additional Practice

1. Sandra's scores on the first 4 tests were 87, 92, 76, and 89.

   a. What is the minimum score Sandra needs to make on the fifth test so that her mean test score is at least 85? Explain.

   b. What is the minimum score Sandra needs to make on the fifth test so that her mean test score is at least 80? Explain.

   c. Can Sandra score well enough so that her mean score is 90 or above? Explain.

   d. If Sandra scores 100 on the fifth test, what is her median test score?

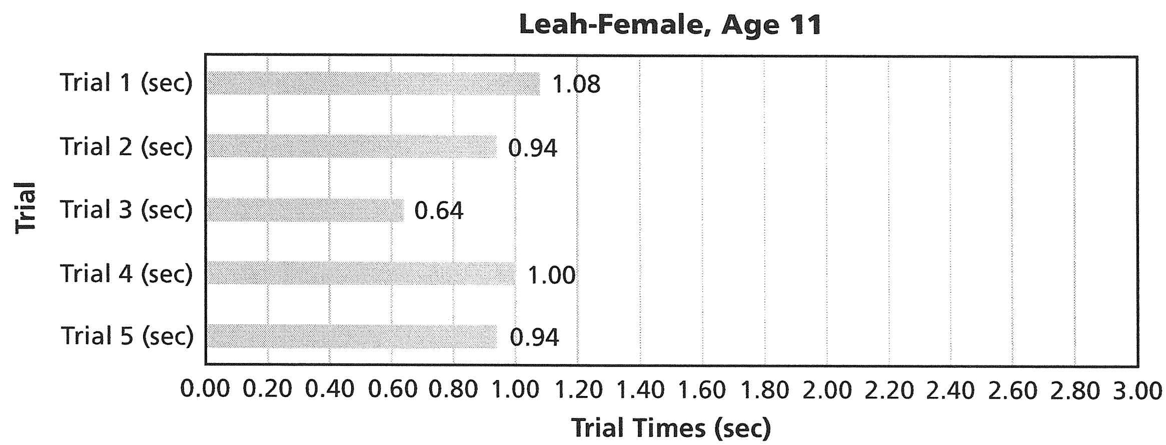
   e. If Sandra scores 0 on the fifth test, what is her median test score?

   f. What score or scores on the fifth test will give Sandra a median test score of 88? 87? 89?

Name ______________________________ Date ____________ Class ____________

Additional Practice

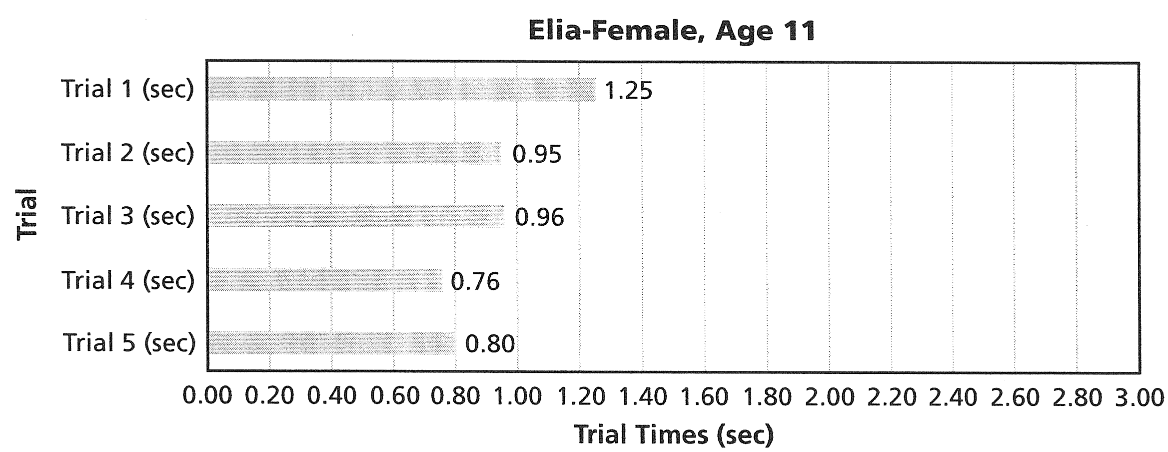
1. Write three different statements that describe the variability in Leah's reaction times from the value bar graph.



2. Below is a value bar graph showing data about Ella's reaction times. Compare Ella's reaction times to Leah's reaction times.
   a. Determine statistics for each student: means, medians, and ranges.

   b. Is one student quicker than the other student? Explain your reasoning.

   c. Is one student more consistent than the other student? Explain.

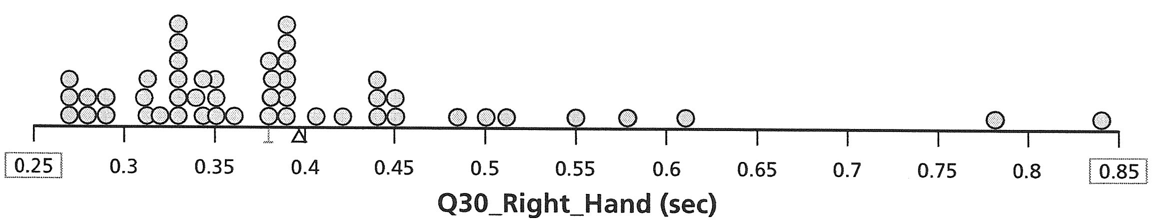


3. The sample of data below is about from 50 students – 25 female students and 25 male students. Two questions on a survey asked students to respond to a stimulus, once with their right hands and once with their left hands. Their time to respond is recorded in seconds. Below are two graphs, one for RIGHT hand and one for LEFT hand response data.

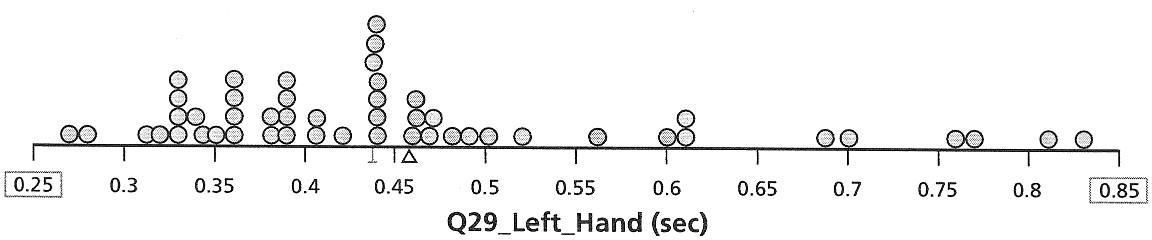
   a. Are students quicker with their right hands or their left hands? Justify your reasoning.

   b. Are students more consistent with their right hands or their left hands? Justify your reasoning.

   c. We have been using data that look at a person's dominant hand and non-dominant hand in the Investigation. Is it possible that, for some of the students, their right hand was their non-dominant hand? Explain.



The mean is 0.39702 sec and the median is 0.38 sec.

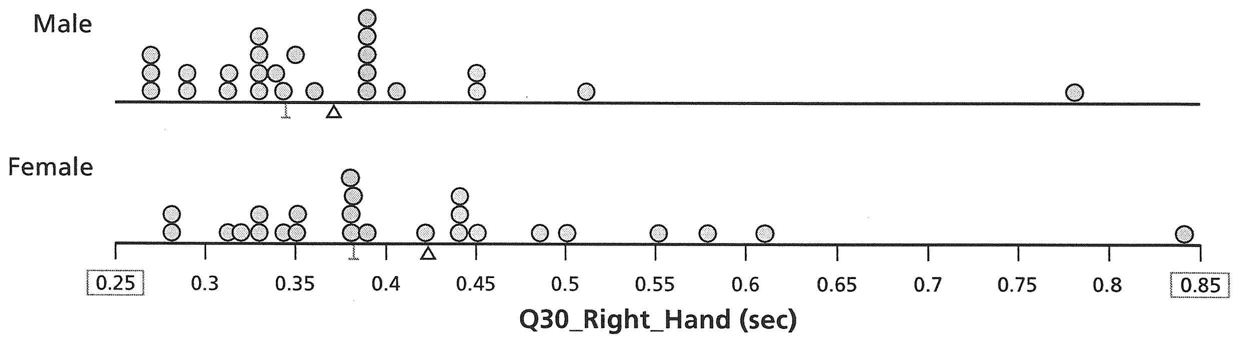


The mean is 0.45726 sec and the median is 0.4375 sec.

SOURCE: www.censusonline.net

Additional Practice *(continued)*

4. Using the same data set about reaction times, compare the male reaction times with their right hands to the female reaction times with their right hands. Look at the graphs below.



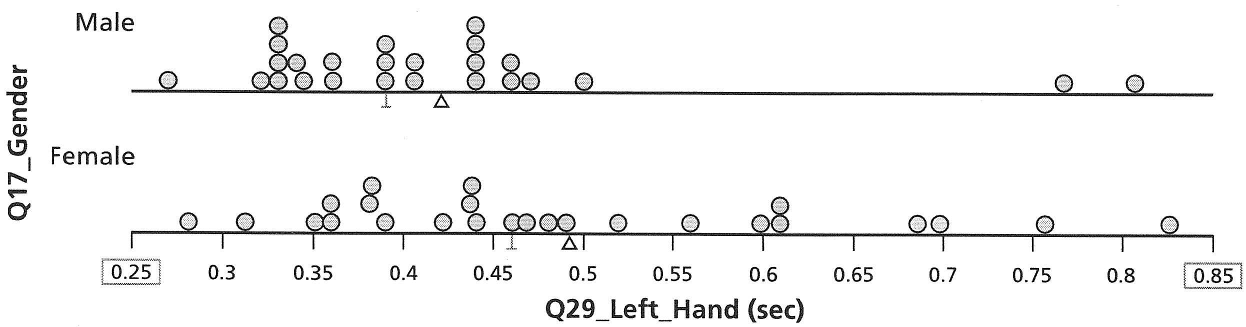
Males: mean = 0.37124 sec and median = 0.344 sec
Females: mean = 0.4228 sec and median = 0.382 sec

a. For both females and males, their means and medians are different. What accounts for this happening?

b. Are females quicker than males using their right hands? Justify your reasoning.

c. Are females more consistent than males using their right hands? Justify your reasoning.

5. Using the same data set about reaction times, compare the male reaction times with their left hands to the female reaction times with their left hands. Look at the graphs below.



Males: mean = 0.42128 sec and median = 0.39 sec
Females: mean = 0.49324 sec and median = 0.461 sec

a. For both females and males, the means and medians are different. What might account for this happening?

b. Are females quicker than males using their left hands? Justify your reasoning.

c. Are females more consistent than males using their left hands? Justify your reasoning.

Name ______________________________ Date ____________ Class ____________

Additional Practice

1. The members of the drama club sold candy bars to help raise money for the school's next play. The stem-and-leaf plot below shows how many candy bars each member of the drama club sold.

Candy Bars Sold by Drama Club

1	0 1 1 1 2 3 5 6 9
2	1 1 1 1 4 7
3	2 3 4 8
4	1 4 9
5	2 3 5 5 8

Key: 3 | 2 means 32 candy bars

a. How many students are in the drama club?

b. How many students sold 25 or more candy bars?

c. How do the numbers of candy bars sold by each student vary?

d. What is the typical number of candy bars sold by each student?

2. Earl rolls 6 six-sided number cubes and finds the sum of the numbers rolled.

a. What are the least and greatest sums Earl can roll? Explain.

b. What do your answers for part (a) tell you about the sums Earl can roll?

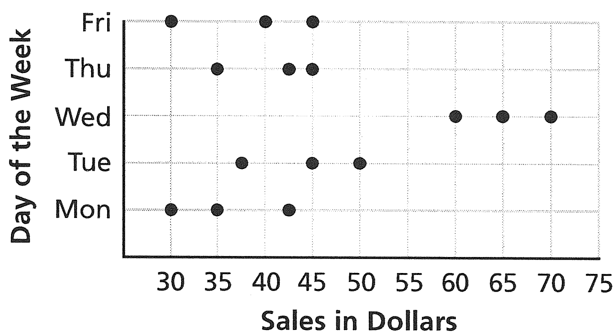
c. Earl rolled the number cubes several times and recorded each sum. Here are Earl's results:

27, 21, 17, 18, 21, 18, 25, 32, 8, 19, 21, 20, 26, 21, 11, 23, 33, 19, 9, 12, 17

Make a stem-and-leaf plot to display Earl's data.

d. Using your stem plot, find the typical sum rolled. Use the median and range to explain your reasoning.

3. Taryn and Travis work in the student store at their school. They made the coordinate graph below to show the total sales each day for three weeks. There are three points corresponding to each weekday because Taryn and Travis recorded their data for the three weeks on a one-week graph.



a. What were the total sales on Tuesdays for the three weeks Taryn and Travis collected their data?

b. Which day of the week seems to be the best for sales at the student store? Explain your reasoning.

c. Which day of the week varies the most for total sales? Explain.

d. How do the sales for the entire three-week period vary?

e. What is the median of the total sales for Fridays? What is the median of the total sales for the three weeks Taryn and Travis collected data?

f. Describe the pattern of sales during a typical week at the student store.

Additional Practice *(continued)*

4. Emily rolled two four-sided number cubes 12 times and computed the sum for each roll. She recorded the results as ordered pairs. The first coordinate is the number of the roll, and the second coordinate is the sum for that roll. For example, (9, 2) means that on her ninth roll Emily rolled a sum of two. The results of Emily's rolls were: (1, 7), (2, 8), (3, 3), (4, 4), (5, 6), (6, 3), (7, 5), (8, 6), (9, 2), (10, 4), (11, 5), (12, 5).

a. Make a coordinate graph of Emily's data. Use the horizontal axis for the number of the roll and the vertical axis for the result.

b. What is the mode of the sums of Emily's rolls? Explain.

c. How do the sums vary?

d. What is the median of the sums? Explain.

e. Does the coordinate graph you made in part (a) show a pattern in Emily's number-cube rolls? Explain.

For Exercises 5–7, use the stem-and-leaf plot below.

Students' Foot Lengths

1	7
2	0 0 0 0 1 1 1 1 1 2 2 2 2 2 2 2 2 3 3 4 4 4 5 5 6 7 7 8
3	0 2

5. How many students are in the class?

A. 3 **B.** 12 **C.** 30 **D.** 33

6. How do the foot lengths for this class vary?

F. 1 to 3 **G.** 7 to 2 **H.** 17 to 32 **J.** 20 to 28

7. What is the median foot length for this class?

A. 2 **B.** 20 **C.** 22 **D.** 24.5

11.5 Go for the Gold!

Stem-and-Leaf Plots

Learning By Doing Lesson Map

Get Ready

Objectives

In this lesson, students will

- Interpret stem-and-leaf plots.
- Create stem-and-leaf plots.

Key Terms

- stem-and-leaf plot

NCTM Content Standards

Data Analysis and Probability Standards Grades 6–8 Expectations

- Find, use, and interpret measures of center and spread, including mean and interquartile range
- Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots

Lesson Overview

Within the context of this lesson, students will be asked to

- Interpret stem-and-leaf plots.
- Create stem-and-leaf plots.
- Find measures of spread based on information in a stem-and-leaf plot.

Students learn a different way to track data. They also build on what they learned in previous lessons on mean, median, mode, and range.

Essential Questions

The following key questions are addressed in this section:

1. What is a stem-and-leaf plot?
2. How do you read a stem-and-leaf plot?
3. How do you create a stem-and-leaf plot?
4. How can you find mean, median, mode, and range on a stem-and-leaf plot?

Warm-Up

Place the following questions or an applicable subset of these questions on the board or project on an overhead projector before students enter class. Students should begin working as soon as they are seated. While students are working on the Warm-Up exercises, you can attend to clerical tasks like taking role or returning student work.

1. Evaluate each numerical expression.

   8.3 + 7.4 + 8.2 — 23.9

   51 + 64 + 35 + 27 — 177

2. Find the mean for each set of numbers. Round your answer to the nearest whole number.

   8.3, 7.4, 8.2 — 8

   51, 64, 35, 27 — 44

Motivator

This lesson is about the summer Olympics. The motivating questions talk about the Olympic games.

Who has watched the Olympics? What are some events in the games? How are they scored? What are the awards for the winners? What other rewards might winners earn? When are the next games and where will they be held?

When introducing this lesson, discuss the different events and awards. The winter Olympics and summer Olympics are held two years apart. Different cities around the globe take turns hosting the games.

Explore Together

Problem 1

Ask for a volunteer to read Problem 1 aloud. After a student has read the problem, ask guiding questions to make sure they understand the problem. Have a student restate the problem in his or her own words.

Guiding Questions

- How is 3 the "stem"?
- What number is the "leaf"?
- What is the greatest number of gold medals the U.S. won?
- How can you use the plot to find the range?
- How do you read the numbers in the plot?

Grouping

Have students work in pairs for this activity.

While students are working, circulate around the classroom to observe their progress.

Common Student Errors

Students may not write the stem-and-leaf plot in order, which will make their median number incorrect. Have students organize each series on a "stem" in numerical order so that they can most easily find the median of the data set.

Whenever we work with data sets, it is helpful to try to "picture" or display the data in a meaningful way in order to "see" some interesting patterns that are not obvious when the data is in a list.

Problem 1

You are planning on following the next Olympics very closely on television. In order to better understand the games, you do some research and list the numbers of gold medals that the United States has won in the Summer Olympic Games in different years.

35, 40, 44, 37, 36, 83, 34, 33, 45, 36, 34, 32, 40,

38, 24, 41, 22, 45, 41, 25, 23, 12, 78, 20, 11

A. Order the data. Then find the mean, median, mode, and range of the data.

The mean is 36.36. The median is 36.

The mode is 34, 36, 40, 41, 45. The range is 72.

B. A **stem-and-leaf plot** is a data display that helps you to see the spread of the data. The *leaves* of the data are made from the digits with the least place value. The *stems* of the data are made from the remaining digits in the greater place values. Each data point is listed once in the plot. Complete the plot. The first data point, 11, is done for you.

0	
1	1 2
2	0 2 3 4 5
3	2 3 4 4 5 6 6 7 8
4	0 0 1 1 4 5 5
5	
6	8
7	3
8	

1 | 1 = 11 medals

C. Be sure to include a key that indicates what the stems and leaves indicate. Complete the key in your stem-and-leaf plot.

See above.

11

Explore Together

Investigate Problem 1

You may wish to give students extra practice reading and finding numbers in the stem-and-leaf plot.

Circulate to be sure students understand how to read and interpret their stem-and-leaf plots.

Key Formative Assessments

- How is the stem-and-leaf plot different from a listing of the numbers?
- How are the listings alike?
- How does the stem-and-leaf plot help you find the mean, median, mode, and range?
- Will the mean always be a number on the plot?

Grouping

Have students work in pairs to answer Questions 1–3. Partner groups will team up with another pair in Question 4 to compare their answers. Have them separate into pairs again to complete Problem 2.

Problem 2

Ask for a volunteer to read Problem 2 aloud. After a student has read the problem, ask guiding questions to make sure they understand the problem. Have a student restate the problem in his or her own words.

NOTES

You may wish to show students the exact height of 147 centimeters.

Because several of the stems may not have any leaves, students may have trouble determining the leaves to use. Help students focus on using common intervals, as they did in previous lessons.

Investigate Problem 1

1. Circle the mean of the data in the stem-and-leaf plot in Problem 1. Draw a square around the median of the data. Place a triangle around the mode of the data.
   Put a square around the first instance of 36; put a circle between 6 and 7 to note the mean is 36.36; put triangles around 34, 36, 40, 41, and 45.
2. Does displaying the data in this form make it easier to "see" any trends or interesting patterns that were not obvious from the list in Problem 1? Use a complete sentence to explain why or why not.
   Yes, the stem-and-leaf plot makes it easier to see the median, mode, and range of the data set because it organizes the information in an easy way to find them.
3. How would you describe the number of gold medals that the United States has won over the years? Use a complete sentence to explain your answer.
   The Unites States has won many gold medals—on average, about 36 every summer.
4. Form a group with another partner team. Compare your answers to Questions 1–3. Be sure that if you have any answers on which you do not agree, you work together to find out why.

Problem 2 Olympic Heights

The heights (in centimeters) of 8 female gymnasts on the U.S. Olympic team are listed below.

145, 147, 152, 152, 152, 155, 155, 170

A. Construct a stem-and-leaf plot of the data. Use only as many boxes as you need. Include a key with your plot.

14	5	7								
15	2	2	2	5	5					
16										
17	0									

15 | 2 = 152 centimeters

B. Circle the mean of the data in the stem-and-leaf plot in Part (A). Draw a square around the median of the data. Place a triangle around the mode of the data.
The mean is 153.5 cm. Put a square between the second and third instance of 152 and draw a triangle around 152.

C. How would you describe the gymnasts' heights? Use a complete sentence to explain your answer.
The gymnasts' heights are mostly between 152 centimeters and 155 centimeters. The most common height is 152 centimeters.

Explore Together

Investigate Problem 2

You will probably want to further explain that the raw scores in diving are combined to form one total combined score for each diver. The actual method involves deleting the highest and lowest score, multiplying by the degree of difficulty, and several other processes before they find the actual score! Actual scores will be in the double-digits for Olympians.

Grouping

Have students work in pairs to answer Questions 1–4. Partner groups will team up with another pair in Question 5 to compare their answers.

Guiding Questions

- Does it look like the judges gave fairly consistent scores?
- Is there another way to plot the scores that might be more useful?
- Did you find any shortcuts for finding any of the measures of the data set?

NOTES

Diving scores are actually taken from seven judges. But since two scores are thrown out each time, there are only five numbers for each dive instead of seven.

You may wish to have students suggest shortcuts for adding the numbers together when finding the mean. The plot lends itself to several!

Investigate Problem 2

1. You want to understand how diving scores work in the Olympics. Judges award raw scores on a scale of 1 to 10. This score is then multiplied by the degree of difficulty of the dive. The raw scores from 5 judges for 6 different dives by a diver in an Olympic trial are shown below.

7.8, 6.5, 6.8, 7.0, 7.5, 8.0, 7.8, 7.9, 8.2, 8.7,

5.5, 5.9, 5.4, 5.5, 5.8, 8.7, 8.8, 8.8, 9.0, 8.9,

7.6, 7.5, 7.0, 7.0, 7.4, 6.6, 6.9, 7.3, 7.2, 7.0

Use the space at the left to order the data. Then construct a stem-and-leaf plot of the data. Include a key with your plot.

Stem	Leaves
5	4 5 5 8 9
6	5 6 8 9
7	0 0 0 0 2 3 $\boxed{}$ 4 5 5 6 8 8 9
8	0 2 7 7 8 8 9
9	0

9 | 0 = 9.0

2. Circle the mean of the data in the stem-and-leaf plot. Draw a square around the median of the data. Place a triangle around the mode of the data.
The mean is $7.\overline{3}$ and the median is 7.35. Draw a triangle around 7.0.

3. Does displaying the data in this form make it easier to "see" any trends or interesting patterns that were not obvious from the list above? Use a complete sentence to explain why or why not.
The plot helps to find the trends much more easily because the numbers are organized together.

4. How would you describe the scores of the diver for the six dives? Use a complete sentence to explain your answer.
The diver scored mostly in the low sevens.

5. Form a group with another partner team. Compare your answers to Questions 1–4. Be sure that if you have any answers on which you do not agree, you work together to find out why. Be prepared to share your stem-and-leaf plot with the entire class.

Wrap-Up

Close

To close your lesson, ask students to answer the following questions. Their answers should help summarize the lesson and the learning for the day. You may want to review the Essential Questions from the Get Ready and/or the Key Formative and Guiding Questions found throughout the Lesson Map.

How does a stem-and-leaf plot help you organize information?

How can you read a stem-and-leaf plot?

How can you use a stem-and-leaf plot to find measures of data?

Another alternative for closing your lesson is using the Open-Ended Writing Task provided on the next page.

Ties to the Cognitive Tutor Software

Students will be working in a variety of software units when they encounter this lesson. Ties between text and software can be made because both software and text include modeling with algebraic expressions, graphing, and examining the relationship between written, numeric, graphic, and algebraic representations.

For more specific information on correlations between the software and text, consult the Software Implementation Guide.

Follow-Up

Assignment

Use Assignment 11.5 in the Student Assignments book. See the Teacher's Resources and Assessments book for answers.

Assessment

See the Assessments provided in the Teacher's Resources and Assessments book for this chapter.

Open-Ended Writing Task

Choose another athlete and track his or her scores using a stem-and-leaf plot. Also give the mean, median, mode, and range of his or her scores.

Reflections

Insert your reflections on the lesson as it played out in class today.

What went well?

____

____

What did not go as well as you would have liked?

____

____

How would you like to change the lesson in order to improve the things that did not go well?

____

____

How would you like to change the lesson in order to capitalize on the things that did go well?

____

____

Are there ways in which you feel the lesson could have been enriched for students?

____

____

Students & Basketball Players

Goals

To assess students' ability to—

- read information from a data display;
- choose a representative statistic and justify the choice;
- use representative values to compare data sets.

Materials

A copy of the blackline master "Students and Basketball Players" for each student

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Activity

To introduce this activity, you can ask the students what they know about the heights of professional basketball players—both male and female. You might ask the students how much taller their favorite basketball player is than the typical middle-grades student. (The typical height will, of course, differ for males and females and for sixth graders and eighth graders.)

Students should know what centimeters are, and they should have measured their heights in centimeters. If the students have reported the basketball players' heights in feet and inches, you should challenge them to convert the measurements to centimeters.

Distribute the copies of the activity sheet for the students to work on individually. You may want them to share their solutions with a partner before the whole-class discussion. The point of this assessment is to determine which students understand that the typical difference in heights is the difference in the typical heights and which students use other comparisons (e.g., a comparison of the greatest values in the two data sets).

Discussion

The students should be able to count the number of data elements in the stem plots (25 in each data set) and to explain that each element represents the height of a different person. Four students have heights of 152 cm, and eighteen basketball players have heights of at least 198 cm. These frequencies are simple counts, so students who cannot answer questions 1–3 may not be able to read the stem plots. You may need to review how a stem plot is constructed (see fig. 1.2).

Some students may answer questions 4 and 5 by identifying specific values: modes (152 cm and 205 cm), medians (151 cm and 203 cm), or means (approximately 149.7 cm and 201.6 cm). For each data set, the values for these numerical summaries are similar, so the students may not see a rationale for preferring one value over another. Other students may give ranges of values for questions 4 and 5. For example, they may say that the typical student is between 150 cm and 158 cm tall and the typical basketball player is between 200 cm and 207 cm tall; one argument for these choices is that they are the ends of the stems with the greatest number of leaves. Students often phrase this explanation, "Most of the data are between these values." It is true that more than half the data for the students are between those values. However, more data values for the basketball players are outside the interval from 200 to 207 than are in it. The use of the word *most*, therefore, is not accurate in this example. A correct phrasing of this idea is that the interval from 200 to 207 contains more values than the interval represented by any other stem. You have to decide whether the time is right to make this point explicit.

The intent of question 6 is to find out if the students will subtract whatever typical values they have identified for the two groups or whether they will focus on other values. Some students are likely to subtract the two extreme values (220 – 171). Some students may discount the value 171 cm in the student data because it is separated from the other data; these students may use 220 and 158 as the extreme values (220 – 158). Some students may also give a range of values for the difference, for example, 10 to 90, arguing that the difference from 170 (the stem with the greatest values of student data) to 180 (the stem with the least values of player data) is 10 and the difference from 130 (the stem with the least values of student data) to 220 (the stem with the greatest values of player data) is 90. Although this argument is correct, it is not very helpful for answering question 6.

Selected Instructional Activities

The following activities provide different contexts in which comparing data sets is important. They are in no particular order, but Classroom Climate involves a more complex data set, so it should probably not be the first task you present to the students. In each of the activities, the students are given two data sets with equal *N*s. The students must then decide how to represent those data to facilitate the comparison of the data sets. For each activity, several representations are acceptable. In the follow-up to each activity, you may want the students to discuss the relative merits of the different representations.

In the activity Batteries, the data for Always Ready batteries span a noticeably larger interval than the data for Tough Cell batteries. This difference may cause some students to be unsure about which representations to use and may influence the inferences that students make. The students' justifications for their choices should help you identify the depth of their understanding of representations and numerical summaries.

Students & Basketball Players

Name ______________________________

How much taller are basketball players than students? Jane and Sam had some data that showed the heights in centimeters of the students in their class and of twenty-five professional basketball players. Jane made stem plots to display these heights.

Heights of Students

13	8 8 8 9
14	1 2 7 7 7
15	0 0 1 1 1 2 2 2 2 3 3 6 6 7 8
16	
17	1

Key: 14 | 7 means 147 cm

Heights of Basketball Players

18	0 3 5
19	0 2 5 7 8 8
20	0 0 2 3 5 5 5 5 7
21	0 0 0 1 4 5
22	0

Key: 19 | 2 means 192 cm

1. How many students are in the class? __________ How many heights of basketball players have been reported? ________________ How can you tell? ____
____
2. How many students are 152 cm tall? __________ How can you tell? ______________________
____
3. How many basketball players are at least 198 cm tall? __________ How can you tell?__________
____
4. What is the typical height of the students? ______ Explain how you arrived at your answer. ________
____
____
____
5. What is the typical height of the basketball players? ________ Explain how you arrived at your answer.
____
____
____
6. About how much taller are the basketball players than the students in this class? __________
Explain how you arrived at your answer.______________________________________
____
____

Batteries

Goals

Identify characteristics (e.g., mean, range, clusters) of data distributions

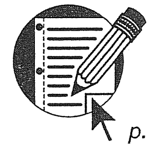
Compare the characteristics of data in order to make decisions

Materials

A copy of the blackline master "Batteries" for each student

Centimeter grid paper (available on the CD-ROM)

A calculator or spreadsheet software



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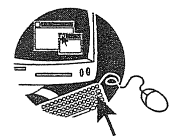


Activity

Distribute a sheet of grid paper and a copy of the activity sheet to each student. Call the students' attention to the data about the life of the two brands of batteries. Since the students did not actually collect the data themselves, it is important for them to think about how the data might have been gathered. Presumably, each battery was put into the same kind of device (e.g., flashlight, boom box), the device was turned on, and a record of the time was kept until the device quit working. The students need to understand that each datum represents the life of a single battery; once the device stops, the battery is dead and cannot be used again.

You can ask questions such as the following to lead the students to think about the conditions under which the device might have been used:

Does ambient temperature affect the life of a battery?

Should the temperature be constant?

If so, does it matter if the constant temperature is very hot or very cold?

Does humidity affect the life of a battery?

Does the time of day affect the life of a battery?

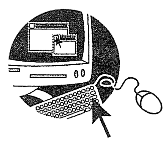
Does the kind of surface on which the device rests affect the life of a battery?

The activity has three parts. The first part (number 1 on the activity sheet) requires the students to think about which representations would be appropriate for displaying these data. The students might use line plots, stem plots, bar graphs, or histograms. One difficulty that students will face in graphing the data for Always Ready batteries is a large range (from 19 hours to 203 hours).

The second part of the activity (numbers 2 and 3 on the activity) requires the students to interpret the data; the representations students create should facilitate their interpretations. The last part (question 4) asks the students to think beyond the data and to use their interpretations to choose the brand of battery that has the longer life.

The students may analyze the data more completely if they work with partners or in small groups. As you observe the groups at work, help the students remain focused on the information in the data sets rather than on personal experiences. For example, some students may comment that they had a calculator "quit" during a test, so Mrs. Brewer should replace the batteries before each test. Although this concern may be important to that student, it is impossible from the data given to make any inferences about how Mrs. Brewer should handle calculator difficulties during tests.

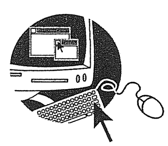
Discussion



Friel and O'Connor (1999) offer ideas that help students compare profiles.

Have several students share their representations and their answers to question 2. Focus the attention of the class on how easy or difficult it is to interpret each of the graphical representations. For example, some students may create line plots, find the median for each data set, and then argue that the brand with the greater median is the better brand of battery. Other students may create graphs but base their argument solely on the computed values of the means, with the greater mean indicating the better brand of battery. In either case, you could ask how the graphs and the computed values show different information about the data.

The graph for Always Ready batteries is J-shaped (see the histogram in fig. 2.5), and the graph for Tough Cell batteries is mound-shaped (see the histogram in fig. 2.6). This difference in the shapes may help some students recognize the difference in the data sets. A solid argument can be made from comparing clusters of data in the two data sets. Line plots or stem plots of the data are other useful representations for revealing clusters (see the back-to-back stem plots in fig. 2.7).



Wilson and Krapfl (1995) discuss the use of technology in similar situations.

The mode and median can be read quickly from the stem plot because the data have been ordered in it. Since each data set has forty entries, the median is located halfway between the twentieth and twenty-first entries. The mean has to be calculated; technology (e.g., a calculator or a spreadsheet) should be used for this task. The summary statistics are shown in table 2.1.

Table 2.1
Summary Statistics for the Battery Data

Statistic	Life of Always Ready Batteries in Hours	Life of Tough Cell Batteries in Hours
Mean	app. 98.0	app. 105.2
Median	105.0	104.5
Mode	115	110
Quartiles	87.5, 105, 113.5	97, 104.5, 110.5

Seventeen of the Always Ready batteries had a battery life greater than or equal to 110 hours, one battery had a life of 175 hours, and one had a life of 203 hours. However, seven of these batteries had a battery life less than 80 hours. Fourteen of the Tough Cell batteries had a battery life greater than or equal to 110 hours. The argument that students may find most compelling is that since the Tough Cell batteries are more consistent, Mrs. Brewer should use that brand.

Some students may draw histograms; the extreme values for the Always Ready batteries may make it problematic to determine the

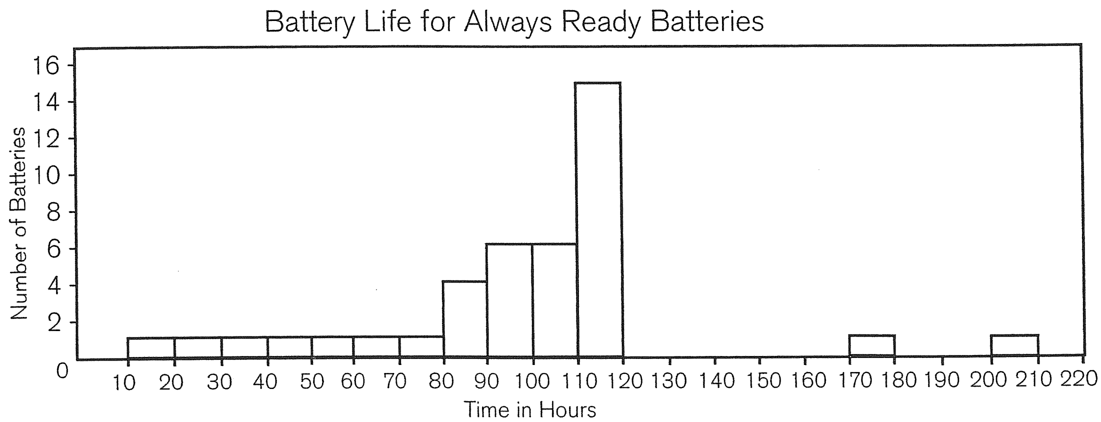


Fig. **2.5.**

A histogram of the data for Always Ready batteries

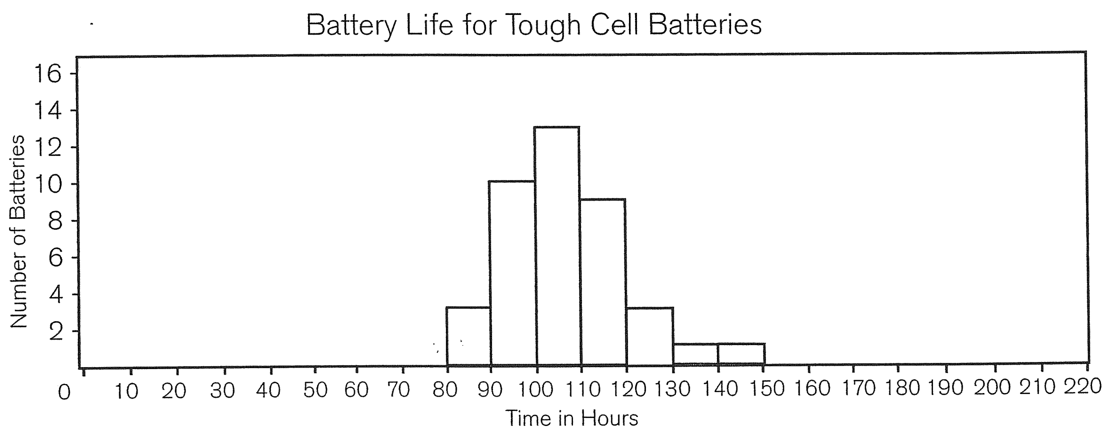


Fig. **2.6.**

A histogram of the data for Tough Cell batteries

Battery Life in Hours for Two Brands

Always Ready Batteries		Tough Cell Batteries
9	1	
2	2	
1	3	
4	4	
2	5	
3	6	
4	7	
9, 6, 6, 4	8	4, 6, 8
9, 6, 6, 5, 3, 2	9	1, 1, 4, 5, 5, 6, 7, 7, 8, 9
7, 6, 5, 5, 1, 1	10	0, 1, 2, 3, 3, 3, 4, 5, 6, 7, 7, 8, 9
7, 6, 5, 5, 5, 5, 5, 4, 3, 1, 1, 0, 0, 0, 0	11	0, 0, 0, 0, 1, 2, 4, 5, 5
	12	1, 1, 1
	13	0
	14	0
	15	
	16	
5	17	
	18	
	19	
3	20	

On the left side, 6 | 8 means 86. On the right side, 8 | 6 means 86.

Fig. **2.7.**

A back-to-back stem plot of the battery data appropriate intervals. It is important that the intervals be the same for the two data sets. Otherwise, it would be very difficult to compare the graphs. Computer software or graphing calculators would make the students' work much easier. With technology, the students can change the scale and the interval until they see a clear and reasonable picture of the distributions. Many students, however, may need to create one histogram on graph paper to have a clear understanding of how to interpret the electronic histograms.

Some students may say that they want "the battery that lasts 203 hours," which would indicate a confusion about what the data represent. Each datum is the number of hours that a particular battery (presumably chosen at random) lasted. That particular battery is dead, and it cannot be used again. There is no way to tell whether another battery, chosen at random, will be a "19 hour" battery or a "203 hour" battery. The point of analyzing these data is to find patterns that will apply to a new sample of batteries, without knowing how any particular battery will perform. The difference between interpreting a single data value (e.g., 203 hours) and interpreting an entire data set is not easy for students to grasp. Opportunities to work with several data sets are required for this idea to be internalized.

By changing the context, you can help the students understand the connection between the data and the context. For example, suppose you were a hiker and you needed a battery that lasts more than 110 hours for your cell phone. Which of these brands would you choose? A greater number of Always Ready batteries lasted more than 110 hours (13 for Always Ready, 10 for Tough Cell), so in this situation, the Always Ready brand might be the better choice.

Extensions

Possible extension activities include having students research actual brands of batteries (e.g., in *Consumer Reports* or on the Internet) or conduct experiments to determine how heat and cold affect battery life.

For further work on data sets with equal *N*s, see "Instructions for Users of Minitools" and Minitool 1, Using Case-Value Plots to Compare Data, on the CD-ROM. Four pairs of data sets for use with Minitool 1 have been supplied on the CD-ROM: Braking Distance, Cotton, Life Span of Batteries, and Watermelon. The context for each pair of data sets is explained in the "Instructions for Users of Minitools." Students use case-value plots to compare two data sets and answer a question about the comparison. Students can also enter their own data sets as files for use with this software. Directions for data entry can be found in the "Instructions for Users of Minitools."

In the activity Batteries, the students compare only two data sets. In the activity Stopping Distances, the students examine two pairs of stopping distances—one for 30 MPH and one for 60 MPH. After examining each pair, the students draw a general conclusion about which model of car seems safer; that is, the students are asked to make two conclusions, each based on data, and then to draw an inference on the basis of the two conclusions. By attending to the language that the students use for these two kinds of activities, you can determine how clearly they understand the differences in the tasks.

Batteries

Name ______________________________

Mrs. Brewer uses graphing calculators in her mathematics classes. She wants to use longer-lasting batteries so that a minimum of time is lost because of dead batteries. She searched the Internet and found the following data on the life, in hours, of two brands of batteries.

Battery Life in Hours

Always Ready Batteries

96	111	110
115	95	115
106	86	110
115	63	84
44	110	52
110	111	107
115	116	89
113	92	93
74	117	101
115	175	101
114	31	105
19	105	99
86	22	
203	96	

Tough Cell Batteries

101	95	103
91	121	106
104	121	111
84	110	114
103	94	107
121	97	99
107	109	96
97	105	110
103	112	110
115	115	98
140	102	108
130	91	110
86	95	
100	88	

1. On grid paper, make two graphical representations that illustrate the differences between these two brands of batteries.
2. What are the quartiles for each set of data? ______________________________

   ____

   What do these values indicate about the two brands? ______________________________

   ____

   ____
3. Which brand of batteries seems to have the longer life? ______________________________

   Explain the reasons for your answer. ______________________________

   ____

   ____
4. Suppose that Mrs. Brewer puts new batteries of the brand you named in question 3 in all the calculators at the beginning of the school year. About when during the year will she need to have replacements available? ______________________________ Explain the reasons for your answer. ______________________________

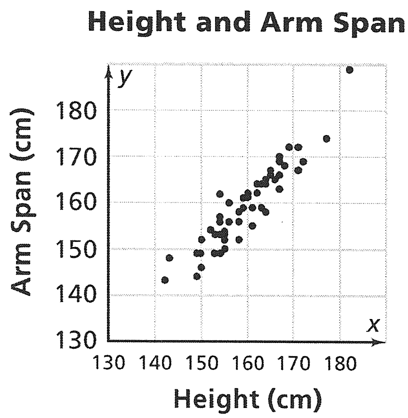
   ____

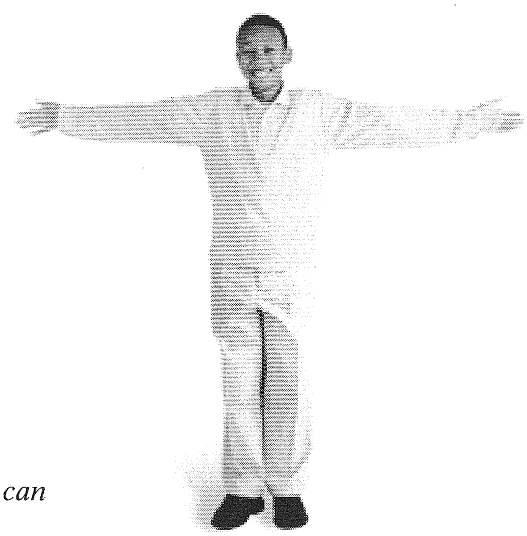
   ____

   ____

4.2 Writing an Equation to Describe a Relationship

In *Data About Us*, a group of 54 sixth-grade students measured their arm spans and their heights. Their data are shown in the scatter plot.





If you know someone's height, what can you say about his or her arm span?

Getting Ready for Problem 4.2

Find a line to model the trend in the data.

- Where does your line cross the y-axis?
- What is the y-coordinate of the point on the line with an x-coordinate of 190?
- What is the x-coordinate of the point with a y-coordinate of 175?

Problem 4.2 Writing an Equation to Describe a Relationship

A. Consider a line through (130, 130) and (190, 190).

1. How might you use this line to describe the relationship between height and arm span?
2. Write an equation for this line using h for height and a for arm span.
3. What is true about the relationship between height and arm span for the points on the line? For the points above the line? For the points below the line?

B. 1. Make a scatter plot of the (*body length, wingspan*) data from the table.

Airplane Comparisons

Plane	Engine Type	Body Length (m)	Wingspan (m)
Boeing 707	jet	46.6	44.4
Boeing 747	jet	70.7	59.6
Ilyushin IL-86	jet	59.5	48.1
McDonnell Douglas DC-8	jet	57.1	45.2
Antonov An-124	jet	69.1	73.3
British Aerospace 146	jet	28.6	26.3
Lockheed C-5 Galaxy	jet	75.5	67.9
Antonov An-225	jet	84.0	88.4
Airbus A300	jet	54.1	44.9
Airbus A310	jet	46.0	43.9
Airbus A320	jet	37.5	33.9
Boeing 737	jet	33.4	28.9
Boeing 757	jet	47.3	38.1
Boeing 767	jet	48.5	47.6
Lockheed Tristar L-1011	jet	54.2	47.3
McDonnell Douglas DC-10	jet	55.5	50.4
Aero/Boeing Spacelines Guppy	propeller	43.8	47.6
Douglas DC-4 C-54 Skymaster	propeller	28.6	35.8
Douglas DC-6	propeller	32.2	35.8
Lockheed L-188 Electra	propeller	31.8	30.2
Vickers Viscount	propeller	26.1	28.6
Antonov An-12	propeller	33.1	38.0
de Havilland DHC Dash-7	propeller	24.5	28.4
Lockheed C-130 Hercules/L-100	propeller	34.4	40.4
British Aerospace 748/ATP	propeller	26.0	30.6
Convair 240	propeller	24.1	32.1
Curtiss C-46 Commando	propeller	23.3	32.9
Douglas DC-3	propeller	19.7	29.0
Grumman Gulfstream I/I-C	propeller	19.4	23.9
Ilyushin IL-14	propeller	22.3	31.7
Martin 4-0-4	propeller	22.8	28.4
Saab 340	propeller	19.7	21.4

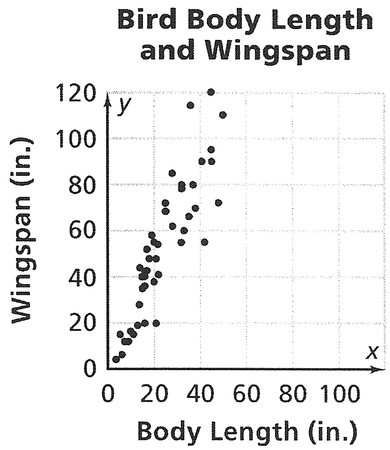
SOURCE: *Airport Airplanes*

2. Does your equation relating height and arm span from Question A also describe the relationship between body length and wingspan for airplanes? Explain.

3. Predict the wingspan of an airplane with a body length of 40 meters.

4. Predict the body length of an airplane with a wingspan of 60 meters.

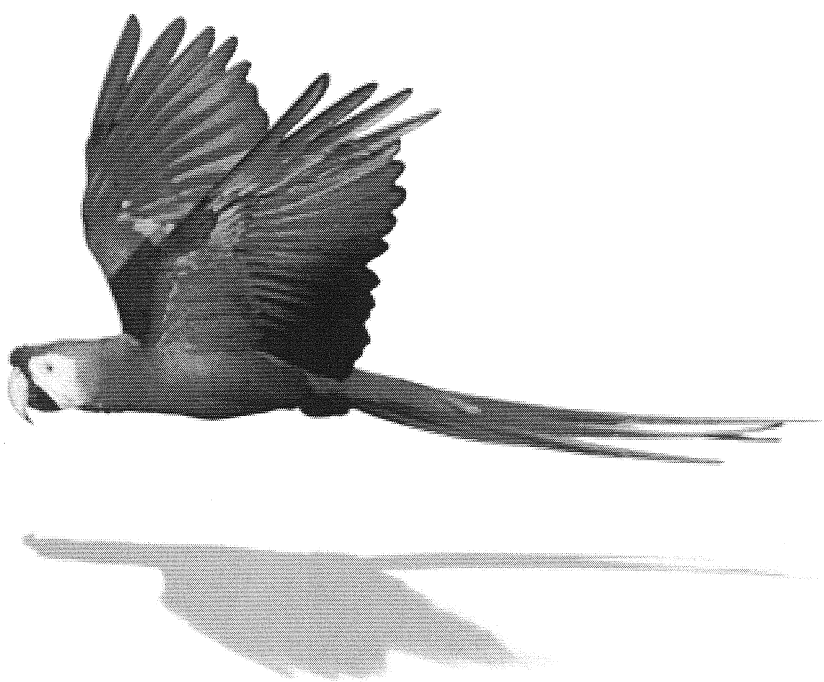
C. **1.** Use the scatter plot below. Does your equation relating height and arm span from Question A also describe the relationship between body length and wingspan for birds? Explain.



2. Find a line that fits the overall pattern of points. What is the equation of your line?

3. Predict the wingspan for a bird whose body length is 60 inches. Explain.

ACE **Homework starts on page 69.**



4.2 Writing an Equation to Describe a Relationship

Goal

- Explore relationships between paired values of numerical attributes

Students explore three different but related proportional relationships: height and arm span for people, body length and wingspan for airplanes, and body length and wingspan for birds. For each, they consider fitting a line to describe the pattern of the relationship and writing an equation to describe this relationship.

Launch 4.2

Introduce the problem as presented in the student edition. Remind students of the activity from *Data About Us* where they looked at the relationship between height and arm span. Even if they do not recall this activity, most students have explored this relationship before.

Have students work in pairs to do the problem.

Explore 4.2

Students first explore the provided scatter plot for height and arm span. They consider the equation for a line to describe this relationship.

Then, they explore a similar relationship for airplanes; they first make the scatter plot using provided data.

Next, they explore a similar relationship for birds; they use the scatter plot provided in the text.

Summarize 4.2

Have students present their solutions and discuss the relationships they found.

At a Glance

PACING 1–2 days

4.2 Writing an Equation to Describe a Relationship

Mathematical Goal

- Explore relationships between paired values of numerical attributes.

Launch

Introduce the problem as presented in the student edition. Remind students of the activity from *Data About Us* where they looked at the relationship between height and arm span. Even if they do not recall this activity, most students have explored this relationship before.

Have students work in pairs to do the problem.

Materials
- Transparencies 4.2A, 4.2B

Explore

Students first explore the provided scatter plot for height and arm span. They consider the equation for a line to describe this relationship.

Then, they explore a similar relationship for airplanes; they first make the scatter plot using provided data.

Next, they explore a similar relationship for birds; they use the scatter plot provided in the text.

Materials
- Labsheet 4.2 (optional)
- Graphing calculators
- Grid paper

Summarize

Have students present their solutions and discuss the relationships they found.

Materials
- Student notes
- Overhead graphing calculator (optional)

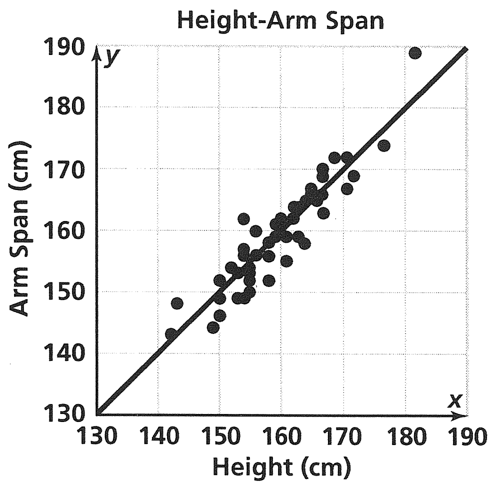
ACE Assignment Guide for Problem 4.2

Core 2, 10–13
Other *Connections* 9, 27–30; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 9: *Data Distributions;* 10–13: *Thinking With Mathematical Models*

Answers to Problem 4.2

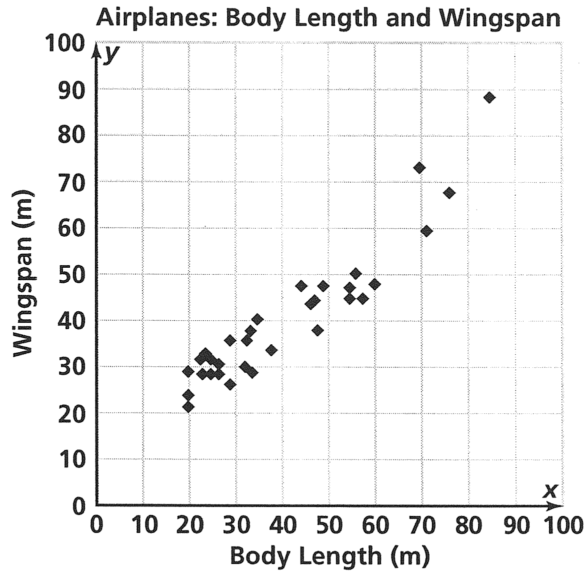
A. 1. Possible answer: As height increases, arm span increases as evidenced by the data clustering around the diagonal from (130, 130) to (190, 190).



2. height $=$ arm span or $h = a$.

3. height $=$ arm span (points on the line)
height $<$ arm span (points above the line)
height $>$ arm span (points below the line)

B. 1.



2. Yes, $y = x$ seems to be a good way to describe this relationship. The points cluster around $y = x$.

3. A reasonable estimate for the wingspan of an airplane with a body length of 40 meters is between 35 and 45 meters

4. A reasonable estimate of the body length of an airplane with a wingspan of 60 meters is between 60 and 65 meters

C. 1. No. The slope of the line that fits the data is about 2 so $y = 2x$ would be the line. If students look at the silhouettes of the birds, it is easy to see that wingspan is about twice body length.

2. Possible answer: $y = 2x$

3. 120 inches; Using the equation $y = 2x$ where x is the body length of a bird and y is the wingspan, a bird with a body length of 60 inches would have a wingspan of 120 inches.

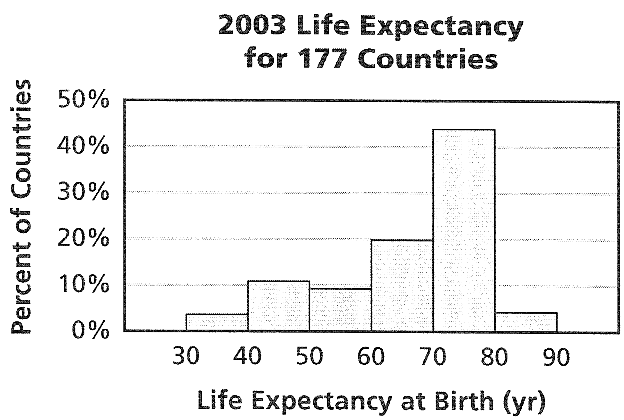
4.3 Human Development Index and Life Expectancies

The Human Development Index (HDI) is a number used to report how well a country is doing in overall human development. The HDI measures the average achievement in three basic dimensions of human development—a long and healthy life, access to education, and a decent standard of living.

Countries with an HDI of over 0.800 are part of the high human development group. Countries from 0.500 to 0.800 are part of the medium group. Countries below 0.500 are part of the low group.

Problem 4.3 Analyzing a Relationship

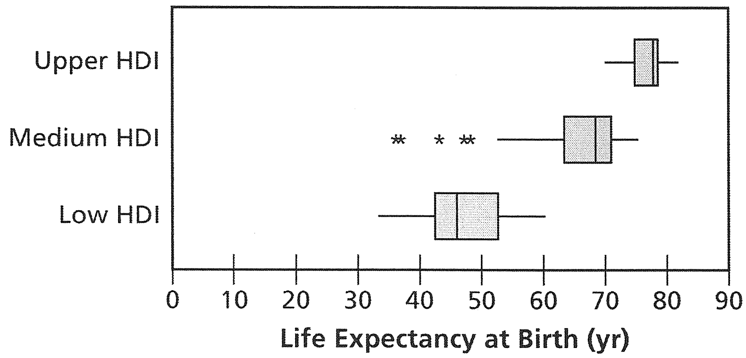
A. 1. Describe the variability in the data in the histogram.



2. Estimate the percent of the countries with life expectancies of 60 years or greater.

3. Use the box plots. Describe how the life expectancies of the countries with upper and medium HDIs compare with the life expectancies of countries with low HDIs.

2003 Life Expectancy and HDI for 177 Countries



4. The medium HDI group has outliers. Using the table, identify which countries are the outliers. Explain.

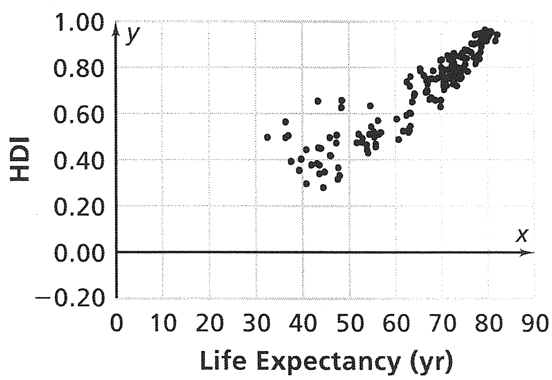
2003 Life Expectancy and HDI

Country	Life Expectancy at Birth (yr)	HDI
Lao People's Dem. Rep.	54.7	0.545
Botswana	36.3	0.565
Zimbabwe	36.9	0.505
South Africa	48.4	0.658
Equatorial Guinea	43.3	0.655
India	63.3	0.602
Namibia	48.3	0.627
Uganda	47.3	0.508

SOURCE: *United Nations Development Programme*

B. Use a straightedge to locate the line $y = 0.01325x - 0.166$ on the scatter plot shown below. **Hint:** Use the equation to find two points, $(0, y_1)$ and $(80, y_2)$, on the line.

2003 Life Expectancy and HDI for 177 Countries



SOURCE: *United Nations Development Programme*

1. How well does this line model the relationship between life expectancy and HDI?
2. Use this line to estimate the HDI for $x = 90$ years.
3. Describe how you can use this line to estimate HDI when you know life expectancy.

ACE **Homework starts on page 69.**

4.3 Human Development Index and Life Expectancies

Goal

- Explore relationships between paired values of numerical attributes

Students explore the relationship between safe water and life expectancies for several different countries in the world. This relationship is not like the relationships in Problem 4.2.

Launch 4.3

Introduce the problem as presented in the student edition. Make sure that the students understand the table of data. Using a world map, help students locate the various countries listed on the table. Discuss what safe water means (drinkable water; also tied to this is appropriate sanitation practices). Discuss what might be the reasons that some countries would have less access to safe water than other countries.

Have students work in pairs to do the problem.

Explore 4.3

Students explore the data, making two different kinds of representations.

Summarize 4.3

Have students present their solutions and discuss the relationships they found.

4.3 Human Development Index and Life Expectancies

At a Glance

PACING 1 day

Mathematical Goal

- Explore relationships between paired values of numerical attributes.

Launch

Introduce the problem as presented in the student edition. Make sure that the students understand the table of data. Using a world map, help students locate the various countries listed on the table. Discuss what safe water means (drinkable water; also tied to this is appropriate sanitation practices). Discuss what might be the reasons that some countries would have less access to safe water than other countries.

Have students work in pairs to do the problem.

Materials
- Transparency 4.3

Explore

Students explore the data, making two different kinds of representations.

Materials
- Graphing calculators
- Computers and statistical software (optional)
- Grid paper

Summarize

Have students present their solutions and discuss the relationships they found.

Materials
- Student notes
- Overhead graphing calculator (optional)

ACE Assignment Guide for Problem 4.3

Differentiated Instruction
Solutions for All Learners

Core 3, 14–26
Other *Connections* 31–35; *Extensions* 36; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.
Connecting to Prior Units 24: *Moving Straight Ahead, Thinking with Mathematical Models*

Answers to Problem 4.3

A. 1. The range of life expectancy is from about 30 to 90 years. About half of the data falls in the interval of 70 to 80 years while the majority of the rest are spread out through the three intervals between 40 and 70 years. A small percentage falls on the intervals of 30-40 and 80-90 years.

2. The percent of countries with life expectancies of 60 years or greater is approximately 75%.

3. Life Expectancies of countries with Upper and Medium HDI levels are much greater than those with low HDI levels. There is also less variability in Life Expectancy for countries with Upper HDI levels. The range is 70–80 years, and half of the population has a life expectancy of about 72–77 years. Countries with lower HDI levels have lower typical life expectancies and also much less consistency.

4. Botswana, Zimbabwe, South Africa, Equatorial Guinea, Namibia, and Uganda are the countries that are outliers because all of their life expectancies fall below the furthest point on the Medium HDI boxplot.

B. 1. The data clusters around this line. The line models the relationship well.

2. $y = 0.01325(90) - 0.166 = 1.0265$

3. If you know life expectancy, then you can find that value on the horizontal axis and draw a vertical line. This line intersects the modeling line $y = 0.01325x - 0.166$ at a point with a y-coordinate of the HDI for the given life expectancy.

Reading a Scatterplot

Goals

To assess students'—

- skill in identifying the two values associated with each point in a scatterplot;
- ability to interpret trends in data in a scatterplot;
- ability to use trends to make predictions.

Materials

- A copy of the blackline master "Reading a Scatterplot" for each student.

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Activity

This activity explores the relationship between the populations of states and the number of telephone area codes in the states. Begin by asking, "Why do phone companies add area codes to telephone numbers?" The students should be able to figure out that the reason is that the phone companies begin to run out of seven-digit phone numbers to assign to customers. Ask, "Why do phone companies begin to run out of phone numbers to use?" The students may know that the shortage is a result of a greater use of regular phones, cellular phones, fax machines, and communication lines for data or computers. Ask, "Why is there a greater demand for phone lines?" The students may say that the demand is due to more businesses and to an increasing population. The activity focuses on population as a variable that is correlated with the number of area codes in the states.

Distribute a copy of the activity sheet to each student. The students should work with a partner to help enrich their interpretation of the data. As you observe the pairs working, encourage them to give clear explanations of the reasoning behind their answers.

Discussion

The numbers of area codes in the states are discrete data—for example, there cannot be 3.5 areas codes in a state—so the graph on the activity sheet may seem somewhat unusual to students. Questions 1 and 2 familiarize students with reading data points on a scatterplot by asking them to read information directly from the graph. For question 1, the students need to count the number of dots above the value 5 on the horizontal axis. For question 2, the students have to interpret the two values associated with point *A*.

Questions 3, 4, and 5 ask the students to make predictions or interpolations from the data. Questions 3 and 5 ask for an interpolation within the range of values of the data, and question 4 asks for predictions that go beyond the range of the data. The students may find an approximate line of fit to make their predictions. Through oral discussion or the students' written explanations, you should be able to discern whether the students have some sense of the linear trend of the data. That is, their predictions should not be just wild guesses; they should be supported by such arguments as "As the population increases by x million people, the number of area codes seems to increase by y."

Question 6 assesses the students' ability to describe the relationship between the two variables. The point of the question is not to elicit an exact answer. Rather, the responses will allow you to assess whether the students can use their observations logically to formulate a description of the relationship that makes sense (both to you and to other students). For example, to begin to make sense of the trend, some students may focus on how to find a representative value for the stack of dots displayed above each value on the horizontal axis (number of area codes). One strategy is to pick the value in the middle of these dots; another strategy is to pick a value in the greatest concentration of the stack of dots. More information about students' responses to these questions can be found in Mooney (2002).

Selected Instructional Activities

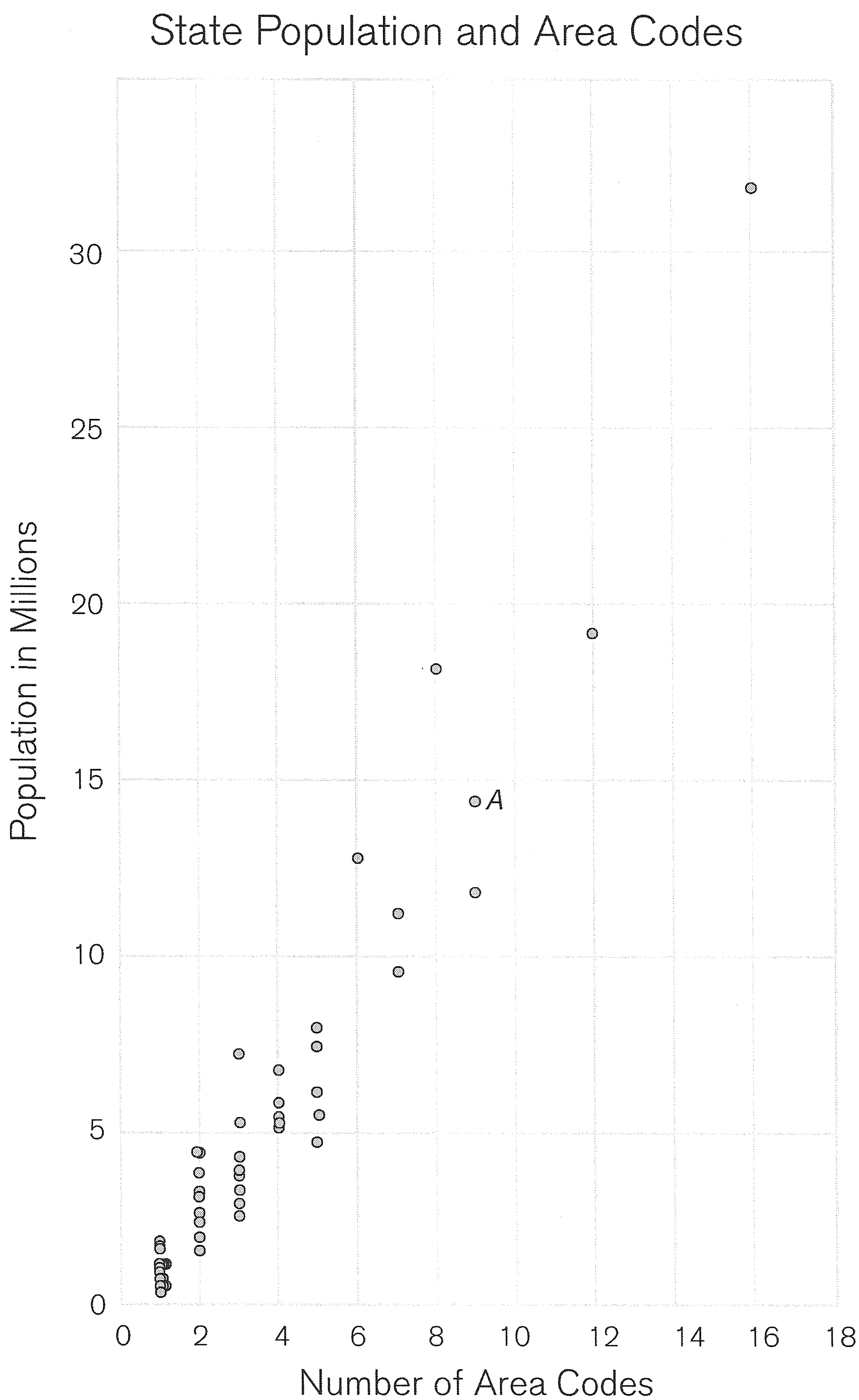
The following activities afford students several opportunities to create scatterplots and then to identify and describe trends in the data. That is, the activities help students explore whether two variables covary and if so, what the nature of that covariation is. Learning how to make scatterplots is one goal of these activities, but it is not the main one. It is more important for students to learn to interrogate bivariate data, whether presented in tables or in graphs, just as they learned to interrogate univariate data from the activities in the first three chapters.

Students must go beyond simply reading data from a scatterplot; they need to learn how to represent bivariate data. They do so in the activity Congress and Pizza, but they may not elect to use a scatterplot. If they do not, then discussing the students' other representations can help you learn how the students are thinking about bivariate data.

Reading a Scatterplot

Name ______________________________

The graph below shows the population and the number of area codes for each state in the United States. Use the scatterplot to answer the questions on the next page.



Reading a Scatterplot (continued)

Name ______________________________

1. How many states have five area codes? __________________ How did you determine the number?
____
____

2. What does the point labeled "A" represent? ______________________________________
How did you decide what it represented? ____
____

3. The population of Canada is approximately 29,100,000. If a similar pattern exists there, how many area codes do you think Canada has? __________ Explain your answer.______________________
____
____

4. The population of Great Britain is approximately 58,600,000. If Great Britain used the same system, how many area codes do you think it would need? ______ Explain your answer. ____________
____
____

5. Suppose that a state had fourteen area codes. What do you think the population of that state would be? ________________ Explain your answer. ______________________________________
____
____

6. Describe the relationship between a state's population and the number of area codes assigned to it.
____
____
____
____
____

Migraines: Histograms

Goals

- Use histograms and relative-frequency histograms to analyze data
- Extract information from a histogram and a relative-frequency histogram

Materials

p. 94

- A copy of the blackline master "Migraines: Histograms" for each student
- Calculators (optional)
- Half-centimeter grid paper, available on the CD-ROM

Activity

To introduce this activity, ask the students what they know about how new drugs are tested. Once a new drug is ready to be tested in humans, it is likely to be tested with only a small number of patients. If it is being compared with a drug already in use, much more data will be available for the approved drug. Data for the two drugs will, therefore, have dramatically different *N*s. For this activity, assume that drug A is a traditional drug that has been approved and used for some time and that drug B is the new drug. This scenario explains the large discrepancy between the numbers of data points in the two data sets.

This activity demonstrates to students the difference in the utility of an absolute-frequency histogram and a relative-frequency histogram. The relative-frequency histogram allows a direct comparison from graphs of data sets that would otherwise be difficult to compare because they contain unequal *N*s. If the students do not know how to make a histogram, instruct them in the construction of histograms either as part of this activity or as a prelude to it.

The scenario presented on the blackline master indicates that each patient records her or his own information. You might want to raise the issue of self-reporting and ask the students whether they think this method might affect the accuracy of the information. Distribute grid paper and a set of activity sheets to each student. Calculators, if available, will help the students compute the relative frequencies. The students should work with a partner or in a small group to encourage rich discussions.

Discussion

Some students are likely to compare these distributions by choosing a "cut point" and counting the number of values in each distribution above or below that cut point. Suppose, for example, that you were interested in knowing how many patients in each group received relief within sixty minutes. Thirty-six patients took drug A and received relief in sixty minutes or less, but only thirty-one patients who took drug B received relief in sixty minutes or less. The students might conclude that drug A is faster acting. This direct, additive comparison is faulty, however, because the numbers of patients using the drugs differ dramatically. A better strategy is to find the percent of the patients who received relief within sixty minutes. About 34 percent of the patients taking drug A and about 66 percent of those taking drug B received relief within that time. The percent for drug B is almost twice that for drug A.

Doing analysis with cut points, however, has some disadvantages. The most important is that often no rationale can be given for the use of any particular cut point. Choosing sixty minutes, for example, seems intuitively reasonable, since it is an hour and people might expect to receive relief within an hour. But is sixty minutes a better (or worse) cut point than sixty-four minutes or fifty-three minutes or even seventy-seven minutes? Without knowing something about the physiology of migraine headaches, we probably cannot make a medically and statistically appropriate choice. A cut point, then, is likely to be completely arbitrary, at least from a mathematical point of view, and different cut points can often lead to different conclusions. Another disadvantage is that for each cut point selected, the percent of data values above or below it must be recomputed. These disadvantages can be overcome somewhat with the use of a relative-frequency histogram. The following is an example of how one teacher led a discussion of this activity that helped her students reason about the proportions of data in particular intervals.

Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken

Teacher: In a test conducted at a major hospital, people who suffered from migraine headaches were given one of two drugs. Drug A has been on the market for several years and has been found to provide relief to many people who suffer from migraine headaches. Drug B is a newer drug that has the potential to give faster relief to more people. The drugs were given to people with migraine headaches, and the subjects were asked to record the amount of time it took them to get relief after taking the drug. We have results from 106 people who took drug A and 47 people who took drug B.

Doug: Why didn't they do the same number of people for each drug? That would be more fair.

Teacher: Why do you think they don't have the same number of results for each drug?

Montez: Because they probably gave it to the same number of people but just got the results after, say, a month, and that's how many people had headaches in that time.

Teacher: Very nice, Montez. This brings out a very important fact about comparing data sets, and that is that they are often not the same size. Will that make a difference in how you conduct your analysis? You might want to think about that.

The students looked at the data and tried to answer question 1 on the activity sheet. The teacher brought them together for a brief period of sharing.

Kyle: We found out how many patients in each group got relief in less than forty minutes. We found that 22 patients who took drug A got relief in less than forty minutes and only 20 who took drug B, so we chose drug A.

Andrea: We did the same thing, but we used thirty minutes and we got 12 with drug A and only 9 with drug B.

Shade: Wow, so either way, drug A is better!

At this point, the students were focused only on the data in the range of zero to thirty or forty minutes. Although their direct, additive comparisons did tell them how many patients received relief in a short time, the students were ignoring an important aspect of the analysis. They were not reasoning about the fraction, or the percent, of the patients that fell in the range from zero to thirty or forty minutes. The teacher suggested that they make the histogram and relative-frequency histogram "to see if those representations change your mind." After students have constructed the displays, it is important to discuss the different features of the graphs, which this teacher did:

Teacher: What conclusions can you draw about the effectiveness of the two drugs by comparing the relative-frequency histograms that you have just completed?

Kyra: Well, it looks like drug A has the highest percentage of people at the high end of the graph.

Teacher: Can someone else say what they think Kyra is saying? Jose?

Jose: She is saying that if you look at the graph of drug A, then it has its highest bars, which mean the largest percentage of patients, at the high end, meaning it took more time to get relief.

Teacher: So, on the basis of what Kyra and Jose said, would you recommend drug A?

Dante: No, you want drug B because you want your tall bars, or your high percentages, to be low, where it takes less minutes to get relief.

Teacher: Yeah, but I thought drug A had some high bars in the low numbers. I mean, look at twenty to forty minutes. They have a bunch of people here.

Dante: Yeah, but it's a very low percentage of all the people that took drug A, so you really don't have a very good chance of getting relief quick with drug A like you do with drug B.

The importance of this discussion is that it helped the students understand what information can be extracted from a comparison of the relative-frequency histograms. Discussions of activities, such as this conversation, are an essential part of helping students make sense of and improve their reasoning.

The following activity, Migraines: Box Plots, extends the analysis of the drug data. Learning how to construct a box plot is one of the goals of this activity, but it is more important that students understand how the box plots help them interpret the data. Box plots are particularly valuable for interpreting the spread of data.

Migraines: Histograms

Name ______________________________

Below are data collected from patients who suffer from migraine headaches. The patients were instructed to take their assigned drugs as soon as their headaches began and to record how much time passed before the drugs gave relief. Drug A is a traditional drug, and Drug B is an experimental drug. Each value is the number of minutes (rounded to the nearest two minutes) that elapsed before a patient got relief.

Drug A (106 patients)

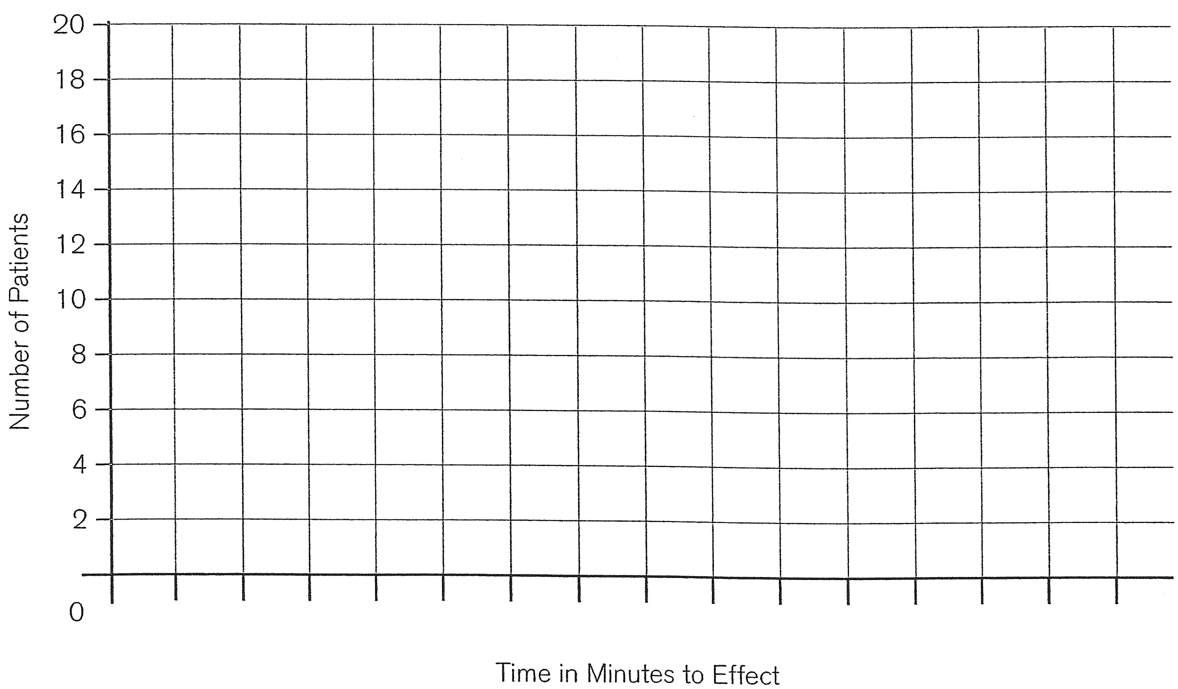
16, 18, 18, 20, 22, 22, 24, 24, 26, 26, 28, 28, 30, 30, 32, 32, 34, 36, 36, 36, 38, 38, 40, 42, 44, 44, 46, 46, 48, 50, 54, 56, 56, 58, 58, 58, 62, 62, 64, 64, 66, 68, 68, 70, 70, 70, 72, 72, 74, 76, 76, 76, 78, 78, 80, 80, 80, 82, 82, 84, 84, 84, 86, 86, 88, 88, 88, 88, 90, 90, 90, 90, 90, 92, 92, 92, 92, 94, 94, 94, 96, 96, 98, 98, 98, 98, 100, 100, 100, 100, 102, 102, 102, 104, 104, 106, 106, 108, 108, 108, 110, 110, 112, 114, 118, 120

Drug B (47 patients)

18, 20, 20, 22, 24, 24, 24, 26, 26, 30, 30, 30, 34, 34, 34, 36, 36, 36, 38, 38, 40, 40, 44, 44, 46, 50, 52, 52, 56, 56, 58, 62, 62, 66, 74, 74, 78, 88, 94, 98, 98, 100, 104, 106, 110, 116, 120

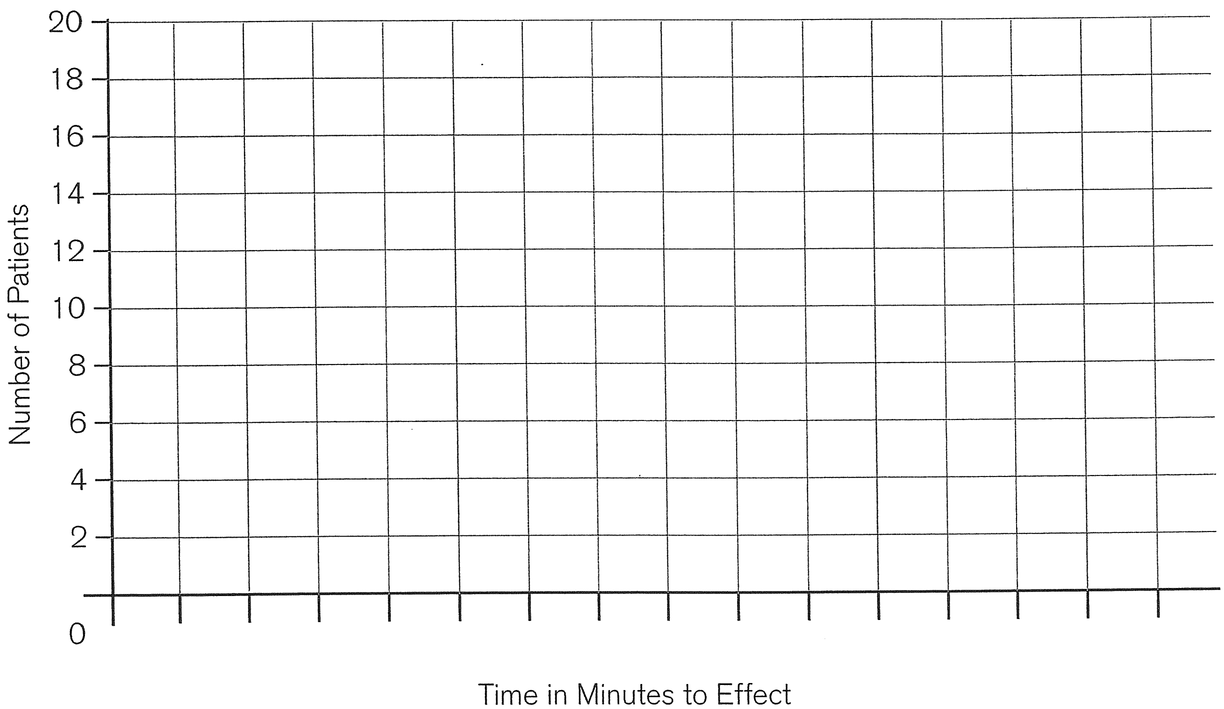
1. From examining these data, which drug do you think gave faster relief from headache pain? __________
   Explain. ____
   ____
   ____

2. Construct a histogram for each data set on the axes below. Title your display, and specify an appropriate scale on the horizontal axis.



Migraines: Histograms (continued)

Name ________________________________

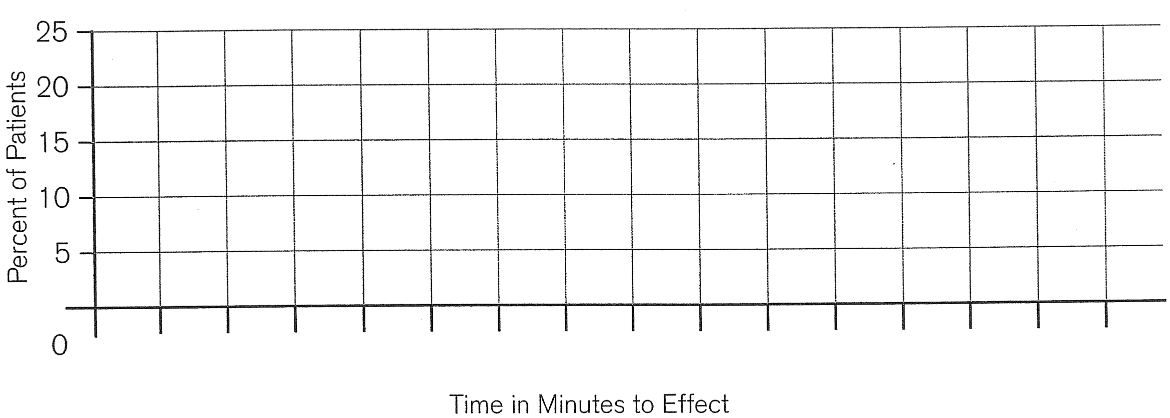


3. From examining the histograms, which drug do you think was more effective in giving fast relief from headache pain? ________ Explain. ____

____

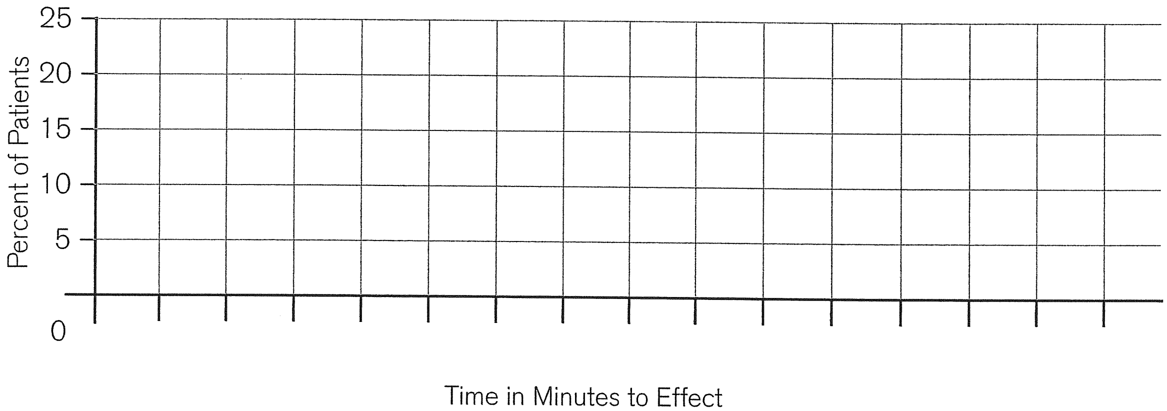
____

4. Construct a relative-frequency histogram on the axes below.



Migraines: Histograms (continued)

Name ______________________________



5. On the basis of your examination of the relative-frequency histograms, which drug do you think gave faster relief from headache pain? ______________ Explain. ______________________________
____
____

6. Some students get different answers for questions 3 and 5. Why do you think that happens?________
____
____

7. How does changing the display change the information you can read from the graph?
____
____
____

8. What advantage does the histogram have over the relative-frequency histogram?____________
____
____

9. What advantage does the relative-frequency histogram have over the histogram?____________
____
____
____

Migraines: Box Plots

Goals

- Practice creating box plots
- Extract information from box plots and use that information to make decisions

Materials

- A copy of the blackline master "Migraines: Box Plots" for each student.

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Activity

The context and data for this activity are the same as for Migraines: Histograms. The focus here is on making box plots, extracting information from the box plots, and then drawing conclusions. Distribute a set of activity sheets to each student. The students may make faster progress if they work with a partner.

Discussion

The range of fifty percent of the data can be determined by looking at the median and one of the extremes. For example, for drug A, the lower 50 percent of the data are between sixteen minutes and seventy-eight minutes; for drug B, the lower 50 percent of the data are between eighteen minutes and forty-four minutes. These statistics present a rather compelling argument in favor of drug B, since half the patients received relief in forty-four minutes or less.

Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatterplots

Locating the quartiles helps make a stronger argument. The quartiles are the upper limit of the lower 25 percent and the lower limit of the upper 25 percent of the data. In conjunction with the median and the extremes, the quartiles divide the data into four groups, each of which contains 25 percent of the data. These values form the five-point summary (see table 3.2).

Table 3.2
The Five-Point Summary for the Drug Data

	Least Value	First Quartile	Median	Third Quartile	Greatest Value
Drug A	16 min	46 min	78 min	94 min	120 min
Drug B	18 min	30 min	44 min	74 min	120 min

For drug A, 25 percent of the data are forty-six minutes or less, whereas for drug B, 50 percent of the data are forty-four minutes or less. For drug A, 50 percent of the data are seventy-eight minutes or less, whereas for drug B, 75 percent of the data are seventy-four minutes or less. Drug B seems clearly to provide faster relief for a greater percent of patients. We can imagine the same teacher who led the classroom discussion for the previous activity continuing with a discussion of this activity with box plots:

Teacher: OK, using the graphs you just made that show the five lines on the axis, what conclusions can you draw about which drug is faster?

Sarah: I think that drug B is faster.

Teacher: What is the basis of your decision, Sarah?

Sarah: OK. Well, there are two groups in drug B below forty-four minutes and only one group in drug A below forty-six.

Teacher: So what does that tell me if I get migraine headaches?

Juan: I know! Half the people who took drug B got relief in forty-four minutes or less, but only one-fourth of the people who took drug A got relief in forty-six minutes or less.

Sharika: There's another way to look at it.

Teacher: OK, Sharika, what is that?

Sharika: Well, you could also say that 50 percent of the people who took drug A had to keep hurting for at least seventy-eight minutes, but only 25 percent of the people who took drug B had to hurt that long.

Teacher: Hmm, so who is correct? Juan or Sharika?

Students: Both.

Teacher: But how can that be?

Jamie: 'Cause it just depends on how you want to say it. There's lots of ways that are right as long as you understand the graph.

Teacher: OK, let's look at the two graphs for the two drugs. Notice where the line for the median is located in each box. What does that tell us about the data?

Megan: In drug A, the second 25 percent of the data are more spread out than in drug B.

Teacher: That's right. The space between the end of the box and the median line tells us the range of that quarter of the data.

Kyle: Yeah, so if we know that 25 percent of the data are in each interval, then the ones that are close together have the data bunched up, so that should be where the clump or cluster of the data is.

Many students have trouble reading a box plot. They interpret it to mean that the farther apart the bars are the more data the interval includes. It is important to emphasize the idea that 25 percent, or one-fourth, of the data fall in each interval. This idea can be reinforced by writing "25%" at the top of each interval.

It is important to stress in different ways what can be inferred about the distribution of the data from the width of the intervals. The interval width does not indicate the frequency of the data in the interval. Each interval contains 25 percent of the data. The interval width indicates how clustered or spread out the data are.

McClain, Cobb, and Gravemeijer (2000) describe how seventh-grade students used Minitool 2 in an experimental statistics unit.

Extensions

It is interesting to give students a box plot and ask them what the data might look like. Obviously, many different distributions can generate the same box plot, so this task could lead to a lively discussion about which data distribution is the most believable.

For further work using dot plots to compare data sets with unequal *N*s, see "Instructions for Users of Minitools" and Minitool 2 on the CD-ROM. The sets of data to be used with Minitool 2 Using Dot

Plots to Compare Data, that are already on the CD include AIDS, Ambulance, Cardiovascular, Cholesterol Level, Corn, Flu Shots, Migraine, Recycling, Speed Trap, and Weight. Students can use dot plots to compare two different data sets in different contexts (e.g., data about several times for two different ambulance companies to reach destinations) and answer a question about the comparison. The different contexts are explained in "Instructions for Users of Minitools." Students can also enter their own data sets as files to use with this software. Data-entry directions can be found in "Instructions for Users of Minitools."

Conclusion

All the activities in this chapter involve data sets with unequal *N*s. The contexts in which such comparisons occur are common in the real world. Typically, these contexts are also complex, so students need time and experience to develop the kind of reasoning required to analyze such complex data. Sophisticated reasoning is also required in situations in which multiple measurements are made on each case—that is, in situations involving bivariate data. Students can explore bivariate data in the activities in chapter 4.

Migraines: Box Plots

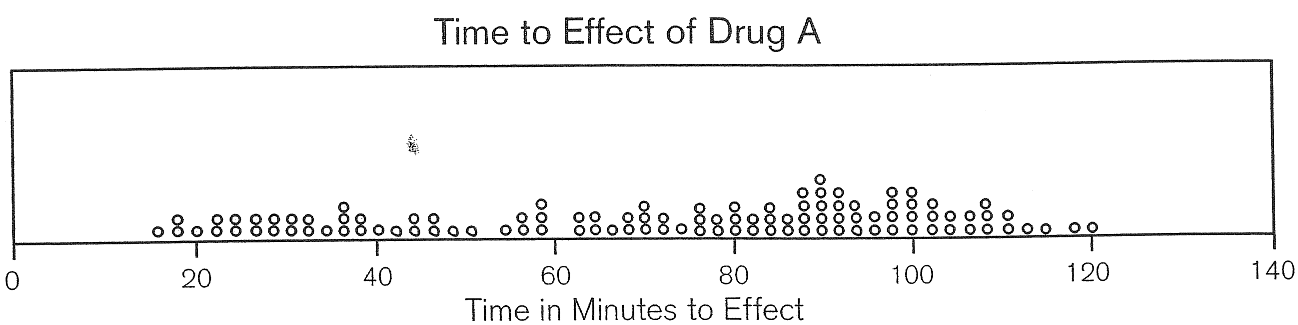
Name ______________________________

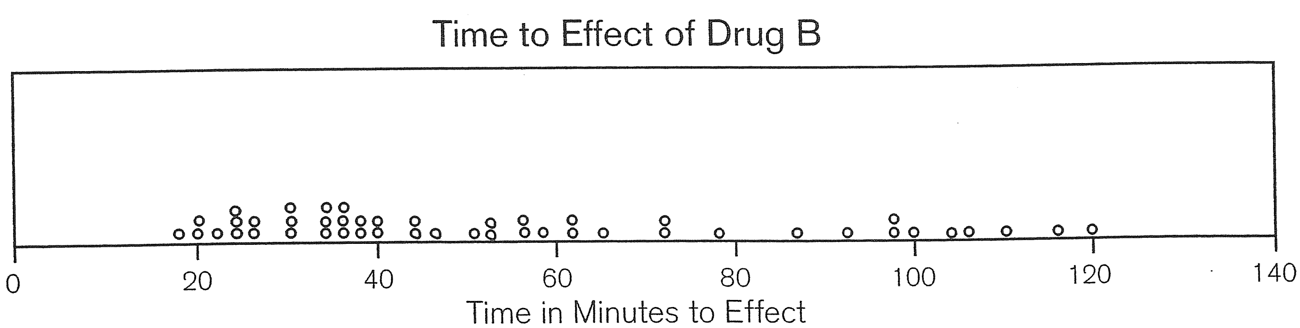
The data below show the number of minutes that elapsed before patients, some taking drug A and some taking drug B, got relief from migraine headaches.

Drug A (106 patients)
16, 18, 18, 20, 22, 22, 24, 24, 26, 26, 28, 28, 30, 30, 32, 32, 34, 36, 36, 36, 38, 38, 40, 42, 44, 44, 46, 46, 48, 50, 54, 56, 56, 58, 58, 58, 62, 62, 64, 64, 66, 68, 68, 70, 70, 70, 72, 72, 74, 76, 76, 76, 78, 78, 80, 80, 80, 82, 82, 84, 84, 84, 86, 86, 88, 88, 88, 88, 90, 90, 90, 90, 90, 92, 92, 92, 92, 94, 94, 94, 96, 96, 98, 98, 98, 98, 100, 100, 100, 100, 102, 102, 102, 104, 104, 106, 106, 108, 108, 108, 110, 110, 112, 114, 118, 120

Drug B (47 patients)
18, 20, 20, 22, 24, 24, 24, 26, 26, 30, 30, 30, 34, 34, 34, 36, 36, 36, 38, 38, 40, 40, 44, 44, 46, 50, 52, 52, 56, 56, 58, 62, 62, 66, 74, 74, 78, 88, 94, 98, 98, 100, 104, 106, 110, 116, 120

One way to represent these data is with dot plots like those below.





1. Use a vertical line segment that intersects the horizontal axis to mark the median of each data set.
2. What do you know about the number of data values on either side of the median? ______________________________

____

____

Migraines: Box Plots (continued)

Name ______________________________

3. On the basis of a comparison of only the medians, which drug appears to provide faster relief? ________ Explain your answer. ____

____

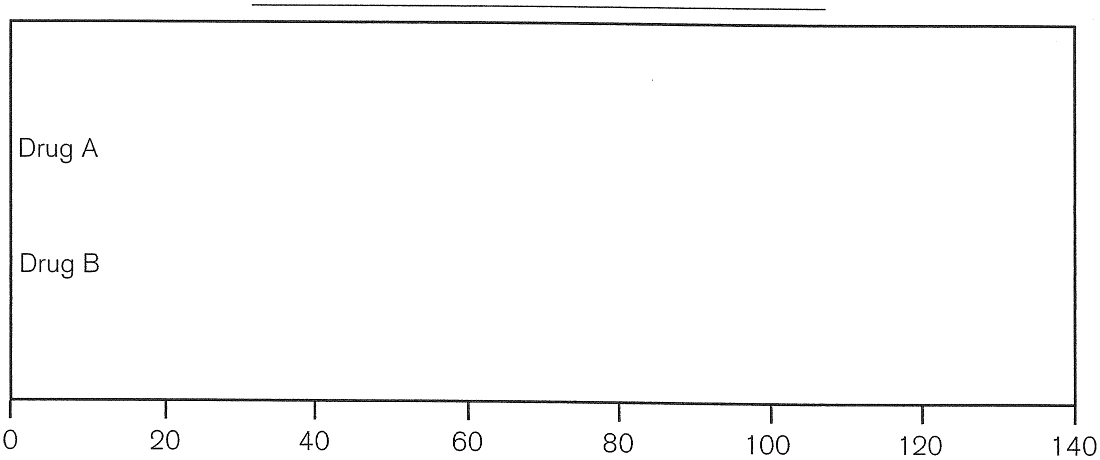
____

4. Use a vertical line segment that intersects the horizontal axis to mark the median of each half of each set of data. You should now have three vertical line segments drawn on the dot plot for each data set. The two new values that you have identified are called the *lower quartile* (or *first quartile*) and the *upper quartile* (or *third quartile*).
5. What do you know about the number of data elements in each of the four intervals for drug A? ________ ____________________ For drug B? ____________________
6. Look again at the lower quartiles, the medians, and the upper quartiles that you marked on the dot plots. On the basis of a comparison of these three values in the data sets, which drug appears to provide faster relief? ______________________ Explain your answer.

____

____

7. Use a vertical line segment to mark each extreme (least and greatest) value in each data set. You should now have five values marked in each data set. These five values are called the *five-point summary* of a data set.
8. Use the five-point summaries and the axis below to make two box plots. Label the axis and title your display.



9. What can you say about the data for drug A in the first interval compared with the data for drug A in the fourth interval? ____

____

Name ______________________________

10. What can you say about the data for drug B in the first interval compared with the data for drug B in the fourth interval? ______________________________

____

____

11. What does the distance between the vertical line segments tell you about how the data are spread out?

____

12. The lower quartile for drug A is about the same as the median for drug B. What does that information tell you about how the speeds of the drugs compare? ______________________________

____

____

13. The median for drug A is about the same as the upper quartile for drug B. What does that information tell you about how the speeds of the drugs compare? ______________________________

____

____

On the basis of all the information, which drug seems to provide faster relief? ______________

Explain your answer. ______________________________

____

____

This plot shows the number of hours students at a middle school spent doing homework one Monday. Use the plot for Exercises 22–24.

Minutes Spent on Homework

Grade 6		Grade 8
000000	0	0
555555	1	055
500	2	0005555
555	3	0000555
550	4	000555
00	5	05
	6	055
0	7	5
5	8	0

Key: 5 | 3 | 0 means 35 minutes for Grade 6 and 30 minutes for Grade 8

22. Find the median homework time for each grade.

23. a. For each grade, describe the variability in the distribution of homework times.

b. Use statistics to explain how the times for sixth-graders compare to the times for eighth-graders.

24. Could these data be used to describe what is typical of all school nights in each of the two grades? Explain.

25. Consider the following data set: 20, 22, 23, 23, 24, 24, and 25.

a. Find the mean and the range of the values.

b. Add three values to the data set so that the mean of the new data set is greater than the mean of the original data set. What is the range of the new data set?

c. Add three values to the original data set so that the mean of the new data set is less than the mean of the original data set. What is the range of the new data set?

d. How do the ranges of the three data sets compare? Why do you think this is so?

Growing, Growing, Growing

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	**6-8 Performance Expectations**
Introducing Exponents **Worksheets**	1	binder		**8.4.A Represent numbers in scientific notation, and translate numbers written in scientific notation into standard form.** **8.4.B Solve problems involving operations with numbers in scientific notation and verify solutions.** **8.4.C Evaluate numerical expressions involving non-negative integer exponents using the laws of exponents and the order of operations.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
(CMP2) Problem 5.2	1	binder or CMP2 Disk		
Investigating Exponents **Worksheet**	1	binder		
Zero and Negative Exponents **Worksheet**	1	binder		
(CMP2) ACE questions pg. 64 #15-41, **(CMP2)** Inv. 5 Skills:Simplifying Exponential Expressions **(CMP2)** Inv. 5 Additional Practice #1, 2, 4 **(CMP2)** Inv. 1 Skill:Using Exponents #1-8	2	binder or CMP2 Disk		
Let's Practice Exponents **Worksheet**	1	binder		
Quiz	1	Not in binder		
Introducing Large Numbers in Scientific Notation **Worksheet**	1	binder		
More w/ Scientific Notation: Large and Small Numbers **Worksheet**	1	binder		
<u>On-line Lesson Exponents and Scientific Notation NO-m</u>	1	binder		
Adding and Subtracting using Scientific Notation **Worksheet**	2	binder		
Multiplying and Dividing using Scientific Notation **Worksheet**	2	binder		
Scientific Notation: Operations Practice **Worksheet**	2	binder		
Quiz	1	Not in binder		
Review for unit test	1			
Unit Assessment	1			
Total Instructional Days	**20**			

Contents in Growing, Growing, Growing

Name: ___________________________Date: _____________ Period: ________

Introducing Exponents

A shorter way of writing long product strings of the same factor such as 2 x 2 x 2 x 2 x 2 x 2 x 2, you can use **exponential form.** For example, you can write 2 x 2 x 2 x 2 x 2 x 2 x 2 as 2^7, which is read "2 to the seventh power."

In the expression 2^7, 7 is the **exponent** and 2 is the **base.** When you evaluate 2^7, you get $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$. We say that 128 is the **standard form** for 2^7.

1. Write each expression in exponential form.

a. 2 x 2 x 2

b. 5 x 5 x 5 x 5

c. 1.5 · 1.5 · 1.5 · 1.5 · 1.5 · 1.5 · 1.5

2. Write each expression in standard from.

a. 2^5

b. 3^3

c. 4.2^3

3. Most calculators have a ^ or a y^x key for evaluating exponents. Use your calculator to find the standard form for each expression.

a. 21^5

b. 3^{10}

c. 1.5^{20}

4. Explain how the meanings of 5^2, 2^5, and 5 x 2 differ.

5. Write each expression in exponential form and standard form.

a. 6 x 6 x 6 x 6

b. 2.5 x 2.5 x 2.5 x 2.5 x 2.5 x 2.5

6. Write each expression in standard form.

a. 2^{10}

b. 10^2

c. 3^9

The following expressions are written in expanded form. Write each one exponential form.

2 x 2 x 2 x 2	10 · 10 · 10 · 10 · 10	2.5 x 2.5 x 2.5 x 2.5
3 x 3 x 3 x 3 x 3 x 3	6 x 6 x 6	1 x 1 x 1 x 1 x 1
-5 x -5 x -5 x -5	3.2 · 3.2 · 3.2	60 x 60 x 60 x 60 x 60
4 x 4 x 4 x 4 x 4 x 4 x 4	-2 · -2 · -2 · -2 · -2	5 · 5 · 5 · 5 · 5

The following expressions are written in exponential form. Write each expression in expanded form, then write them in standard form.

2^{10}	10^{2}	3^{9}
10^{6}	6^{10}	100^{2}
2^{4}	3^{5}	4^{3}
2^{8}	1^{5}	0^{3}

Name: Key Date: ____________ Period: _______

Introducing Exponents

A shorter way of writing long product strings of the same factor such as $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, you can use **exponential form.** For example, you can write $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ as 2^7, which is read "2 to the seventh power."

In the expression 2^7, 7 is the **exponent** and 2 is the **base.** When you evaluate 2^7, you get $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$. We say that 128 is the **standard form** for 2^7.

1. Write each expression in exponential form.

a. $2 \times 2 \times 2$ 2^3

b. $5 \times 5 \times 5 \times 5$ 5^4

c. $1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5 \cdot 1.5$ 1.5^7

2. Write each expression in standard from.

a. 2^5 $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

b. 3^3 $3 \cdot 3 \cdot 3$

c. 4.2^3 $4.2 \times 4.2 \times 4.2$

3. Most calculators have a ^ or a y^x key for evaluating exponents. Use your calculator to find the standard form for each expression.

a. 21^5 4,084,101

b. 3^{10} 59,049

c. 1.5^{20} 3325.25673

4. Explain how the meanings of 5^2, 2^5, and 5×2 differ.

$5^2 = 5 \cdot 5$ OR $\boxed{25}$

$2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ OR $\boxed{32}$

$5 \times 2 = \boxed{10}$

5. Write each expression in exponential form and standard form.

a. $6 \times 6 \times 6 \times 6$ 6^4 or 1296

b. $2.5 \times 2.5 \times 2.5 \times 2.5 \times 2.5 \times 2.5$ 2.5^6 or 244.140625

6. Write each expression in standard form.

a. 2^{10} 1024

b. 10^2 100

c. 3^9 19683

The following expressions are written in expanded form. Write each one exponential form.

2 x 2 x 2 x 2 2^4	10 · 10 · 10 · 10 · 10 10^5	2.5 x 2.5 x 2.5 x 2.5 2.5^4
3 x 3 x 3 x 3 x 3 x 3 3^6	6 x 6 x 6 6^3	1 x 1 x 1 x 1 x 1 1^5
-5 x -5 x -5 x -5 -5^4	3.2 · 3.2 · 3.2 3.2^3	60 x 60 x 60 x 60 x 60 60^5
4 x 4 x 4 x 4 x 4 x 4 x 4 4^7	-2 · -2 · -2 · -2 · -2 -2^5	5 · 5 · 5 · 5 · 5 5^5

The following expressions are written in exponential form. Write each expression in expanded form, then write them in standard form.

2^{10} 2·2·2·2·2·2·2·2·2·2 1024	10^2 10·10 $\boxed{100}$	3^9 3·3·3·3·3·3·3·3·3 $\boxed{19683}$
10^6 10·10·10·10·10·10 1,000,000	6^{10} 6·6·6·6·6·6·6·6·6·6 60,466,176	100^2 100·100 $\boxed{10,000}$
2^4 2·2·2·2 16	3^5 3·3·3·3·3 243	4^3 4·4·4 64
2^8 2·2·2·2·2·2·2·2 256	1^5 1·1·1·1·1 1	0^3 0·0·0 0

E. Find the value of a that makes each number sentence true.

1. $a^{12} = 531,441$ **2.** $a^9 = 387,420,489$ **3.** $a^6 = 11,390,625$

F. Find a value for a and values for the missing digits to make each number sentence true. Explain your reasoning.

1. $a^7 = \_\ \_\ \_\ \_\ \_\ 3$ **2.** $a^8 = \_\ \_\ \_\ \_\ \_\ \_\ \_\ \_\ 1$

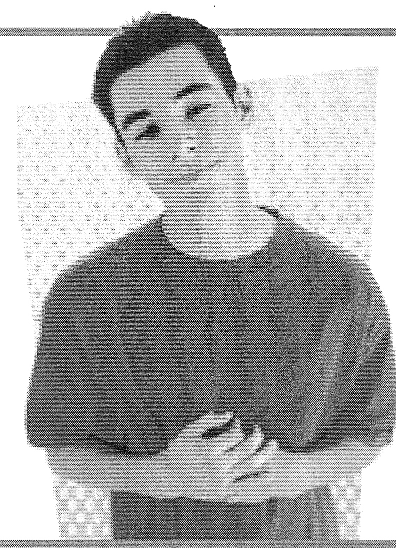
ACE **Homework starts on page 64.**

5.2 Operating With Exponents

In the last problem, you explored patterns in the values of a^x for different values of a. You used the patterns you discovered to make predictions. For example, you predicted the ones digit in the standard form of 4^{12}. In this problem, you will look at other interesting patterns that lead to some important properties of exponents.

Getting Ready for Problem 5.2

- Federico noticed that 16 appears twice in the powers table. It is in the column for 2^x, for $x = 4$. It is also in the column for 4^x, for $x = 2$. He said this means that $2^4 = 4^2$. Write 2^4 as a product of 2's. Then, show that the product is equal to 4^2.
- Are there other numbers that appear more than once in the table? If so, write equations to show the equivalent exponential forms of the numbers.



Problem 5.2 Operating with Exponents

Use properties of real numbers and your table from Problem 5.1 to help you answer these questions.

A. 1. Explain why each of the following statements is true.

a. $2^3 \times 2^2 = 2^5$ **b.** $3^4 \times 3^3 = 3^7$ **c.** $6^3 \times 6^5 = 6^8$

2. Give another example that fits the pattern in part (1).

3. Complete the following equation to show how you can find the exponent of the product when you multiply two powers with the same base. Explain your reasoning.

$$a^m \times a^n = a^{\square}$$

B. 1. Explain why each of the following statements is true.

a. $2^3 \times 3^3 = 6^3$ **b.** $5^3 \times 6^3 = 30^3$ **c.** $10^4 \times 4^4 = 40^4$

2. Give another example that fits the pattern in part (1).

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$$a^m \times b^m = \underline{\ ?\ }$$

C. 1. Explain why each of the following statements is true.

a. $4^2 = (2^2)^2 = 2^4$

b. $9^2 = (3^2)^2 = 3^4$

c. $125^2 = (5^3)^2 = 5^6$

2. Give another example that fits the pattern in part (1).

3. Complete the following equation to show how you can find the base and exponent when a power is raised to a power. Explain.

$$(a^m)^n = \underline{\ ?\ }$$

D. 1. Explain why each of the following statements is true.

a. $\frac{3^5}{3^2} = 3^3$ **b.** $\frac{4^6}{4^5} = 4^1$ **c.** $\frac{5^{10}}{5^{10}} = 5^0$

2. Tom says $\frac{4^5}{4^6} = 4^{-1}$. Mary says $\frac{4^5}{4^6} = \frac{1}{4^1}$. Who is correct and why?

3. Complete the following equation to show how you can find the base and exponent of the quotient when you divide two powers with the same base. (Assume a is not 0.) Explain your reasoning.

$$\frac{a^m}{a^n} = \underline{\ ?\ }$$

E. Use the pattern from Question D to explain why $a^0 = 1$ for any nonzero number a.

ACE **Homework starts on page 64.**

5.2 Operating With Exponents

Goals

- Examine patterns in the exponential and standard forms of powers of whole numbers
- Use patterns in powers to develop rules for operating with exponents
- Become skillful in operating with exponents in numeric and algebraic expressions

Students use the powers table from Problem 5.1 to find special relationships among numbers written in exponential form. For example, students may notice that $4^2 = 2^4$ or $(2^2)^2 = 2^{(2 \times 2)}$. This is an example of a general property of exponents: $(a^m)^n = a^{mn}$. In this problem, students use patterns among exponents to formulate several important properties:

$$(a^m)^n = a^{mn}$$

$$a^m \times a^n = a^{m+n}$$

$$a^m \times b^m = (a \times b)^m$$

$$a^m \div a^n = a^{m-n} \text{ (for } a \neq 0)$$

Launch 5.2

To launch this problem, refer to the completed powers table. Use the Getting Ready to encourage students to begin noticing patterns that will lead to the rules of exponents.

- *Federico noticed that 16 appears twice in the powers table. It is in the column for 2^x, for $x = 4$. It is also in the column for 4^x, for $x = 2$. He said this means that $2^4 = 4^2$. Write 2^4 as a product of 2s. Then, show that the product is equal to 4^2.*
  $[2 \cdot 2 \cdot 2 \cdot 2 = (2 \cdot 2) \cdot (2 \cdot 2) = 4 \cdot 4 = 4^2]$
- *Are there other numbers that appear more than once in the table? If so, write equations to show the equal exponential forms of the numbers.* (Use different colors to circle these numbers. For example, 4 occurs as 2^2 and as 4^1, so $2^2 = 4^1$. Other examples are 8, 64, 9, 81, 256, and 729, 4,096, and 6,561.)

Tell students that in this problem, they will look for a way to generalize these and other patterns for exponents.

Let students work in groups of three or four on this problem.

Explore 5.2

The questions are structured so that most students should be able to see the patterns. Students look at specific cases of each pattern first and are then asked to generalize the patterns.

If students have trouble explaining why a general rule works, have them connect the general rule to a specific case. For example, if students cannot explain why $a^m \times a^n = a^{m+n}$, ask them to first explain why $3^2 \times 3^4 = 3^6$. Students should be able to explain that the product of two 3s and four 3s is the product of two plus four, or six, 3s.

$$\underbrace{3 \times 3}_{\substack{\text{two 3s} \\ 3^2}} \times \underbrace{3 \times 3 \times 3 \times 3}_{\substack{\text{four 3s} \\ 3^4}} = \underbrace{3 \times 3 \times 3 \times 3 \times 3 \times 3}_{\substack{\text{six 3s} \\ 3^{2+4}}}$$

Help them generalize this to $a^m \times a^n$.

$$\underbrace{(a \times a \times \cdots \times a)}_{\substack{m \text{ as} \\ a^m}} \times \underbrace{(a \times a \times \cdots \times a)}_{\substack{n \text{ as} \\ a^n}} = \underbrace{(a \times a \times \cdots \times a)}_{\substack{(m + n) \text{ as} \\ a^{m+n}}}$$

The key to understanding why the rules of exponents work is for students to visualize the structure of a^m as the product of a used m times: $a \times a \times a \times \ldots \times a$.

Part (1) of Question D expresses $\frac{4^5}{4^6}$ as both 4^{-1} and $\frac{1}{4}$. Students may not be familiar with negative exponents. They are discussed in ACE Exercise 54, but students need not fully understand them to complete Question D. The example of $\frac{4^5}{4^6} = 4^{-1}$ illustrates that the general rule $\frac{a^m}{a^n} = a^{m-n}$ holds, even when the result has a negative exponent. Instead of using negative exponents, students can break the rule into two cases:

- If $m \geq n$, then $\frac{a^m}{a^n} = a^{m-n}$ for $a \neq 0$.
- If $m < n$, then $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$ for $a \neq 0$.

Be sure to check on how students are reasoning about a^0.

Summarize 5.2

Ask different groups to present their reasoning for each part of the problem. Use the completed powers table to illustrate the rules. For example, the rule $a^m \times a^n = a^{m+n}$ can be illustrated by looking at any column. The 3^x column is highlighted in Figure 3. Multiply two numbers in this column, for example, $9 \times 81 = 729$. The exponent for 729 is the sum of the exponents for the factors ($3^2 \times 3^4 = 3^{2+4} = 3^6$). Ask students to give other examples. To understand the rules, it is essential that students see $3^2 \times 3^4$ as six 3s multiplied together: $3 \times 3 \times 3 \times 3 \times 3 \times 3$.

This would be a good time to ask students to explain the differences between 3^x, $3x$, and $3 + x$.

The rule $a^n \times b^n = (a \times b)^n$ is illustrated by looking at any row. The row corresponding to a^4 is highlighted in Figure 3. Multiply two numbers in this row, such as $16 \times 256 = 4,096$. In exponential form, this is $2^4 \times 4^4 = 8^4$. Students should think of a string of four 2s followed by four 4s.

The factors can be rearranged to form a string of four (2×4)s, or four 8s.

$(2 \times 2 \times 2 \times 2) \times (4 \times 4 \times 4 \times 4) =$
$(2 \times 4) \times (2 \times 4) \times (2 \times 4) \times (2 \times 4) =$
$8 \times 8 \times 8 \times 8 = 8^4$

Use the table to illustrate the other rules in a similar way.

Check for Understanding

Have students write numeric expressions, such as the following, in simpler exponential form:

$$\frac{2^5 \times 2^6}{2^9} \qquad \frac{3^4 \times 2^6}{6^9}$$

Depending on the goals for your course, you might also ask students to simplify algebraic expressions like these:

$$(x^2)^3 \qquad x^6x^4 \qquad \frac{x^4x^3}{x^7}$$

True or false:

$2^3 \times 2^2 = 6^5$ $\qquad$ $4^2 + 4^3 = 4^5$

$5^3 \times 25 = 5^5$ $\qquad$ $18^4 = 3^4 \times 6^4$

Figure 3

Powers Table

x	1^x	2^x	3^x	4^x	5^x	6^x	7^x	8^x	9^x	10^x
1	1	2	3	4	5	6	7	8	9	10
2	1	4	9	16	25	36	49	64	81	100
3	1	8	27	64	125	216	343	512	729	1,000
4	1	16	81	256	625	1,296	2,401	4,096	6,561	10,000
5	1	32	243	1,024	3,125	7,776	16,807	32,768	59,049	100,000
6	1	64	729	4,096	15,625	46,656	117,649	262,144	531,441	1,000,000
7	1	128	2,187	16,384	78,125	279,936	823,543	2,097,152	4,782,969	10,000,000
8	1	256	6,561	65,536	390,625	1,679,616	5,764,801	16,777,216	43,046,721	100,000,000
Ones Digits of Powers	1	2, 4, 8, 6	3, 9, 7, 1	4, 6	5	6	7, 9, 3, 1	8, 4, 2, 6	9, 1	0

5.2 Operating With Exponents

At a Glance

PACING $1\frac{1}{2}$ days

Mathematical Goals

- Examine patterns in the exponential and standard forms of powers of whole numbers
- Use patterns in powers to develop rules for operating with exponents
- Become skillful in operating with exponents in numeric and algebraic expressions

Launch

Refer to the completed powers table. Use the Getting Ready to encourage students to begin noticing patterns that will lead to the rules of exponents.

Tell students that in this problem, they will look for a way to generalize patterns for exponents.

Let the class work in groups of three or four.

Materials

- Transparencies 5.2A and 5.2B
- Students' completed tables from Problem 5.1

Explore

The questions are structured so that most students should be able to see the patterns. Students look at specific cases of each pattern first and are then asked to generalize the patterns.

The key to understanding why the rules of exponents work is for students to visualize the structure of a^m as the product of a used m times.

Be sure to check on how students are reasoning about a^0.

Summarize

Ask different groups to present their reasoning for each part of the problem. Use the completed powers table to illustrate the rules. For example, the rule $a^m \times a^n = a^{m+n}$ can be illustrated by looking at any column. Multiply two numbers in this column, such as $9 \times 81 = 729$. The exponent for 729 is the sum of the exponents of the factors ($3^2 \times 3^4 = 3^{2+4} = 3^6$). Ask students to give other examples.

This would be a good time to ask about the differences between $3x$, 3^x, and $3 + x$.

Check for Understanding

Have students write numeric expressions, such as the following, in simpler exponential form:

$$\frac{2^5 \times 2^6}{2^9} \qquad \frac{3^4 \times 2^6}{6^9}$$

Depending on the goals for your course, you might also ask students to simplify algebraic expressions like these:

$$(x)^3 \qquad x^6x^4 \qquad \frac{x^4x^3}{x^7}$$

Materials

- Student notebooks

ACE Assignment Guide for Problem 5.2

Differentiated Instruction — Solutions for All Learners

Core 10–27, 31

Other *Applications* 28–30, 32–41; *Connections* 46-48; *Extensions* 56–63; unassigned choices from previous problems

Adapted For suggestions about adapting Exercise 31 and other ACE exercises, see the *CMP Special Needs Handbook.*

Connecting to Prior Units 46, 47: *Filling and Wrapping* and *Stretching and Shrinking*; 48: *Prime Time*

Answers to Problem 5.2

A. 1. Students may calculate each product to verify the equality. For example, in part (a), $2^3 \times 2^2 = 2^5$ is true because $8 \times 4 = 32$. Others may reason that the left side has 2 used as a factor 5 times, which is equal to 32 or 2^5. They may also note that $2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$.

2. Examples will vary.

3. $a^m \times a^n = a^{m+n}$. This is true because the left side of the equality has a as a factor $m + n$ times. Or, some will write out:

$$a^m \times a^n = \underbrace{(a \times a \times \cdots \times a)}_{\substack{m \text{ times} \\ a^m}} \times \underbrace{(a \times a \times \cdots \times a)}_{\substack{n \text{ times} \\ a^n}}$$

$$= \underbrace{(a \times a \times \cdots \times a)}_{\substack{(m+n) \text{ times} \\ a^{m+n}}} = a^{m+n}$$

B. 1. Students may evaluate both sides or use the definition of exponents and the commutative property of multiplication:
$2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3 = (2 \cdot 3)(2 \cdot 3)(2 \cdot 3) = (2 \cdot 3)^3 = 6^3$

2. Examples will vary.

3. $a^m \times b^m = (ab)^m$ Students can generalize the argument in part (1):

$$a^m \times b^m = \underbrace{(a \times a \times \cdots \times a)}_{\substack{m \text{ times} \\ a^m}} \times \underbrace{(b \times b \times \cdots \times b)}_{\substack{m \text{ times} \\ b^m}}$$

$$= \underbrace{(ab \times ab \times \cdots \times ab)}_{\substack{m \text{ times} \\ ab^m}} = ab^m$$

C. 1. Some students will just evaluate both sides. Here is a symbolic argument for part (a):
$4^2 = 4 \times 4 = 2^2 \times 2^2 = (2^2)^2 = 2 \times 2 \times 2 \times 2 = 2^4$

2. Examples will vary.

3. $(a^m)^n = a^{mn}$
Students can think of $(a^m)^n$ *as* a^m used as a factor n times. Each a^m is a string with a used as a factor m times. In all, there are n strings of m as or nm as. Symbolically,

$$(a^m)^n = \underbrace{a^m \times a^m \times \cdots \times a^m}_{n \text{ times}}$$

$$= \underbrace{\underbrace{(a\,a \cdots a)}_{m \text{ times}} \times \underbrace{(a\,a \cdots a)}_{m \text{ times}} \times \cdots \times \underbrace{(a\,a \cdots a)}_{m \text{ times}}}_{n \text{ times}}$$

$$= a^{mn}$$

D. 1. Students can evaluate both sides or write the numerators and denominators as factor strings, and then simplify so there are no common factors in the numerator and denominator.

$$\frac{3^5}{3^2} = \frac{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3} = 3 \cdot 3 \cdot 3 = 3^3$$

$$\frac{4^6}{4^5} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = 4$$

$$\frac{4^5}{4^6} = \frac{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4} = \frac{1}{4}$$

$$\frac{5^{10}}{5^{10}} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5} = 1 = 5^0$$

2. Examples will vary.

3. $\frac{a^m}{a^n} = a^{m-n}$

Some students may claim that $\frac{a^m}{a^n} = a^{m-n}$ will lead to negative exponents if $m < n$. This is an opportunity to define a^{-n} as $\frac{1}{a^n}$.

E. $\frac{a^n}{a^n} = a^{n-n} = a^0$, but $\frac{a^n}{a^n} = 1$, so $a^0 = 1$. This is a subtle point. Some students might say that in the expression a^0, a is used as a factor 0 times, so the product should be 0. Some will use the same argument to say a^0 should be 1. Using the rules provides a logical argument that should help most students. Once a^{-n} is defined as $\frac{1}{a^n}$, the logic becomes a bit clearer.

Name ______________________________ Date__________ Period____

Investigating Exponents:

Same Base and Power to a Power

Identify the parts of the expression: ________ → n^x ← ____________

- The __________ of an exponential expression is the factor being multiplied.
- The ____________ tells us how many times to multiply the base by itself.
- An exponent is sometimes called a __________.

Exponential Expression	**Expanded Product**	**Total Number of Factors**	**Sum of the Exponents (from column 1)**	**Simplified Exponential Expression**
7^5*7^3	(7*7*7*7*7)(7*7*7)	**8** factors	$5+3=8$	7^8
4^2*4^3		☐ factors		
$(-8)^4(-8)^1$		☐ factors		
2^2*2^2		☐ factors		
x^5*x^0		☐ factors		
$(-r)^1(-r)^3$		☐ factors		
y^5*y^2		☐ factors		
$(-10)^3(-10)^3$		☐ factors		
(☺)^1(☺)^1		☐ factors		

In Conclusion:

- Do you notice a pattern in columns 3 and 4? Explain what you notice.
- When two or more factors in an exponential expression have the same base, you can simplify the expression by ______________________________.

POWER TO A POWER

Exponential Expression	1st Expanded Product	2nd Expanded Product	Total Number of Factors	Product of the Exponents	Simplified Exponential Expression
$(7^5)^2$	$(7^5)(7^5)$	(7*7*7*7*7)(7*7*7*7*7)	10	5*2 = 10	7^{10}
$(4^2)^3$					
$((-8)^4)^2$					
$(2^{100})^0$			0	100*0 = 0	**1**
$(x^5)^1$					
$[(-r)^1]^2$					
$(y^1)^1$					
$[(-10)^2]^3$					
$[(☺)^1]^9$					

In conclusion:

- Do you notice a pattern in columns 4 and 5? Explain what you notice.
- An exponential expression when a base raised to a power, is raised to another power, you can simplify the expression by ________________________________.

Name Key Date__________ Period____

Investigating Exponents:

Same Base and Power to a Power

Identify the parts of the expression: base $\rightarrow n^x \leftarrow$ exponent

- The base of an exponential expression is the factor being multiplied.
- The exponent tells us how many times to multiply the base by itself.
- An exponent is sometimes called a power.

Exponential Expression	Expanded Product	Total Number of Factors	Sum of the Exponents (from column 1)	Simplified Exponential Expression
7^5*7^3	(7*7*7*7*7)(7*7*7)	8 factors	5 + 3 = 8	7^8
4^2*4^3	$(4\cdot4)(4\cdot4\cdot4)$	5 factors	2+3=5	4^5
$(-8)^4(-8)^1$	$(-8\cdot-8\cdot-8\cdot-8)(-8)$	5 factors	4+1=5	-8^5
2^2*2^2	$(2\cdot2)(2\cdot2)$	4 factors	2+2=4	2^4
x^5*x^0	$(x\cdot x\cdot x\cdot x\cdot x)(1)$	5 factors	5+0=5	x^5
$(-r)^1(-r)^3$	$(-r)(-r\cdot-r\cdot-r)$	4 factors	1+3=4	$-r^4$
y^5*y^2	$(y\cdot y\cdot y\cdot y\cdot y)(y\cdot y)$	7 factors	5+2=7	y^7
$(-10)^3(-10)^3$	$(-10\cdot-10\cdot-10)(-10\cdot-10\cdot-10)$	6 factors	3+3=6	-10^6
(☺)^1(☺)^1	(☺)(☺)	2 factors	1+1=2	(☺)^2

In conclusion:

- Do you notice a pattern in columns 3 and 4? Explain what you notice.
  when you add the exponents, thats how many factors there are.
- When two or more factors in an exponential expression have the same base, you can simplify the expression by keeping the base and add the exponents

POWER TO A POWER

Exponential Expression	1st Expanded Product	2nd Expanded Product	Total Number of Factors	Product of the Exponents	Simplified Exponential Expression
$(7^5)^2$	$(7^5)(7^5)$	(7*7*7*7*7)(7*7*7*7*7)	10	5*2 = 10	7^{10}
$(4^2)^3$	$(4^2)(4^2)(4^2)$	$(4\cdot4)(4\cdot4)(4\cdot4)$	6	$2\cdot3=6$	4^6
$((-8)^4)^2$	$(-8^4)(-8^4)$	$(-8\cdot-8\cdot-8\cdot-8)\cdot(-8\cdot-8\cdot-8\cdot-8)$	8		-8^6
$(2^{100})^0$	2^{100}		0	100*0 = 0	1
$(x^5)^1$	x^5	$x\cdot x\cdot x\cdot x\cdot x$	5	5	x^5
$[(-r)^1]^2$	$(-r)(-r)$	$(-r)(-r)$	2	$1\cdot2=2$	$-r^2$
$(y^1)^1$	y	y	1	$1\cdot1=1$	y^1
$[(-10)^2]^3$	$(-10^2)(-10^2)(-10^2)$	$-10\cdot-10\cdot-10\cdot-10\cdot-10\cdot-10$	6	$2\cdot3=6$	-10^6
$[(☺)^1]^9$	(☺)'(☺)'(☺)'(☺)'(☺)'(☺)'(☺)'(☺)'(☺)'	☺☺☺☺☺☺☺☺☺	9	$1\cdot9=9$	$(☺)^9$

In conclusion:

- Do you notice a pattern in columns 4 and 5? Explain what you notice.

  you multiply the exponents and it tells you how many factors there are

- An exponential expression when a base raised to a power, is raised to another power, you can simplify the expression by multiplying the exponents and keeping the base the same.

Name ______________________ **Date** ____________ **Period** ______

Zero and Negative Exponents

Quick Review

Use what you know about exponents to write each of the following expressions in expanded form and with a single exponent.

a) $\frac{y^7}{y^2}$ = ______________________ = ________

b) $\frac{3^6}{3^4}$ = ______________________ = ________

c) $\frac{n^{10}}{n^6}$ = ______________________ = ________

Try These

Now, do the same process with these new problems. Write each of the following expressions in expanded form and with a single exponent.

d) $\frac{x^4}{x^4}$ = ______________________ = ________

e) $\frac{a^6}{a^6}$ = ______________________ = ________

f) $\frac{10^3}{10^3}$ = ______________________ = ________

g) $\frac{m^2}{m^5}$ = ______________________ = ________

h) $\frac{c^5}{c^7}$ = ______________________ = ________

i) $\frac{9^4}{9^8}$ = ______________________ = ________

Look at all the problems you have done so far. How can you tell what type of exponent will result simply by looking at the original expression?

Exponent Patterns

This table supports what you have learned about negative exponents and exponents of zero. To go down any column of the table, you divide by 5. Notice that each time you divide, the exponent decreases by 1. Look at the patterns and complete the table.

Exponential Form	Expanded Form	Fraction Form
5^4	$5 \cdot 5 \cdot 5 \cdot 5$	625
5^3		
5^2		
5^1		
5^0		
5^{-1}		
5^{-2}		
5^{-3}		
5^{-4}		

Conclusions

$b^0 =$ ______ $\quad$ $b^{-n} =$ ________ (and $b^n = \frac{1}{b^{-n}}$)

Name ______________________________ **Date** ___________ **Period** ______

Zero and Negative Exponents

Now you try!

Rewrite using a single exponent.

$\frac{a^{14}}{a^{8}} =$

$\frac{d^{7}}{d^{10}} =$

$\frac{5^{6}}{5^{6}} =$

Rewrite using a negative exponent.

$\frac{g^{10}}{g^{13}} =$

$\frac{24^{5}}{24^{10}} =$

$\frac{x^{7}}{x^{3}} =$

Rewrite using a positive exponent.

$\frac{r^{3}}{r^{5}} =$

$\frac{8^{2}}{8^{7}} =$

$\frac{c^{20}}{c^{16}} =$

These problems are MEGA-challenging! Try at least ____ problems. Simplify.

$\frac{a^{5}b^{10}}{a^{3}b^{17}} =$

$\frac{c^{2}d^{7}e}{c^{6}d^{4}e^{4}} =$

$\frac{m^{10}n^{50}}{m^{4}n^{3}} =$

$\frac{81r^{3}}{9r^{5}} =$

$\frac{20x^{2}y^{29}}{4x^{5}y^{20}} =$

$\frac{-10f^{6}}{5f^{11}} =$

$\frac{v^{2}w^{12}}{v^{2}w^{20}} =$

$\frac{jk}{j^{6}k^{3}} =$

$\frac{-15p^{4}}{-3p^{4}} =$

Name ______________________ **Date** __________ **Period** ______

Zero and Negative Exponents

Quick Review

Use what you know about exponents to write each of the following expressions in expanded form and with a single exponent.

a) $\frac{y^7}{y^2}$ = $\frac{y \cdot y \cdot y \cdot y \cdot y \cdot \cancel{y} \cdot \cancel{y}}{\cancel{y} \cdot \cancel{y}}$ = y^5 $7 \cdot 2 = 5$

b) $\frac{3^6}{3^4}$ = $\frac{3 \cdot 3 \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}}{\cancel{3} \cdot \cancel{3} \cdot \cancel{3} \cdot \cancel{3}}$ = 3^2 $6-4=2$

c) $\frac{n^{10}}{n^6}$ = $\frac{\cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot n \cdot n \cdot n \cdot n}{\cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n}}$ = n^4 $10-6=4$

Try These

Now, do the same process with these new problems. Write each of the following expressions in expanded form and with a single exponent.

d) $\frac{x^4}{x^4}$ = $\frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}$ = x^0 or 1

e) $\frac{a^6}{a^6}$ = $\frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}$ = 10^0 or 1

f) $\frac{10^3}{10^3}$ = $\frac{10 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} = 10^0$ = 10^0 or 1

g) $\frac{m^2}{m^5}$ = $\frac{\cancel{m} \cdot \cancel{m}}{m \cdot \cancel{m} \cdot m \cdot m \cdot m}$ = $\frac{1}{m^3}$ or m^{-3}

h) $\frac{c^5}{c^7}$ = $\frac{\cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c}}{\cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot \cancel{c} \cdot c \cdot c}$ = $\frac{1}{c^2}$ or c^{-2}

i) $\frac{9^4}{9^8}$ = $\frac{\cancel{9} \cdot \cancel{9} \cdot \cancel{9} \cdot \cancel{9}}{\cancel{9} \cdot \cancel{9} \cdot \cancel{9} \cdot \cancel{9} \cdot 9 \cdot 9 \cdot 9 \cdot 9}$ = $\frac{1}{9^4}$ or 9^4

Look at all the problems you have done so far. How can you tell what type of exponent will result simply by looking at the original expression?

you subtract the bottom exponent from the top one.

Exponent Patterns

This table supports what you have learned about negative exponents and exponents of zero. To go down any column of the table, you divide by 5. Notice that each time you divide, the exponent decreases by 1. Look at the patterns and complete the table.

Exponential Form	Expanded Form	Fraction Form
5^4	$5 \cdot 5 \cdot 5 \cdot 5$	625
5^3	$5 \cdot 5 \cdot 5$	125
5^2	$5 \cdot 5$	25
5^1	5	5
5^0	1	1
5^{-1}	$\frac{1}{5}$	1/5
5^{-2}	$\frac{1}{5 \cdot 5}$	1/25
5^{-3}	$\frac{1}{5 \cdot 5 \cdot 5}$	1/125
5^{-4}	$\frac{1}{5 \cdot 5 \cdot 5 \cdot 5}$	1/625

Conclusions

$b^0 =$ 1 $\qquad b^{-n} = \frac{1}{b^n}$ (and $b^n = \frac{1}{b^{-n}}$)

Name ______________________ **Date** __________ **Period** ______

Zero and Negative Exponents

Now you try!

Rewrite using a single exponent.

$\frac{a^{14}}{a^{8}} = \boxed{a^6}$ $\frac{d^{7}}{d^{10}} = \boxed{d^{-3}}$ or $\boxed{\frac{1}{d^3}}$ $\frac{5^{6}}{5^{6}} = \boxed{5^0}$

Rewrite using a negative exponent.

$\frac{g^{10}}{g^{13}} = \boxed{g^{-3}}$ $\frac{24^{5}}{24^{10}} = \boxed{24^{-5}}$ $\frac{x^{7}}{x^{3}} = x^4$ $\boxed{\frac{1}{x^{-4}}}$

Rewrite using a positive exponent.

$\frac{r^{3}}{r^{5}} = r^{-2} = \boxed{\frac{1}{r^2}}$ $\frac{8^{2}}{8^{7}} = 8^{-5} = \boxed{\frac{1}{8^5}}$ $\frac{c^{20}}{c^{16}} = \boxed{c^4}$

These problems are MEGA-challenging! Try at least ___ problems. Simplify.

$\frac{a^5b^{10}}{a^3b^{17}} = \frac{a^2}{b^7}$ $\frac{c^2d^7e}{c^6d^4e^4} = \frac{d^3}{c^4e^3}$ $\frac{m^{10}n^{50}}{m^4n^3} = m^6n^{47}$

$\frac{81r^3}{9r^5} = \frac{91}{r^2}$ $\frac{20x^2y^{29}}{4x^5y^{20}} = \frac{5y^9}{x^3}$ $\frac{-10f^6}{5f^{11}} = \frac{-2}{f^5}$

$\frac{v^2w^{12}}{v^2w^{20}} = \frac{1}{w^8}$ $\frac{jk}{j^6k^3} = \frac{1}{j^5k^2}$ $\frac{-15p^4}{-3p^4} = 5$

Applications Connections Extensions

ACE

Applications

Predict the ones digit for the standard form of the number.

1. 7^{100} **2.** 6^{200} **3.** 17^{100} **4.** 31^{10} **5.** 12^{100}

active math online
For: Pattern Iterator
Visit: PHSchool.com
Web Code: apd-3500

For Exercises 6 and 7, find the value of a that makes the number sentence true.

6. $a^7 = 823{,}543$ **7.** $a^6 = 1{,}771{,}561$

8. Explain how you can use your calculator to find the ones digit of the standard form of 3^{30}.

9. Multiple Choice In the powers table you completed in Problem 5.1, look for patterns in the ones digit of square numbers. Which number is *not* a square number? Explain.

A. 289 **B.** 784 **C.** 1,392 **D.** 10,000

Tell how many zeros are in the standard form of the number.

10. 10^{10} **11.** 10^{50} **12.** 10^{100}

Find the least value of x that will make the statement true.

13. $9^6 < 10^x$ **14.** $3^{14} < 10^x$

For Exercises 15–17, identify the greater number in each pair.

15. 6^{10} or 7^{10} **16.** 8^{10} or 10^8 **17.** 6^9 or 9^6

18. Multiple Choice Which expression is equivalent to $2^9 \times 2^{10}$?

F. 2^{90} **G.** 2^{19} **H.** 4^{19} **J.** 2^{18}

Use the properties of exponents to write each expression as a single power. Check your answers.

19. $5^6 \times 8^6$ **20.** $(7^5)^3$ **21.** $\frac{8^{15}}{8^{10}}$

For Exercises 22–27, tell whether the statement is *true* or *false*. Explain.

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22. $6^3 \times 6^5 = 6^8$

23. $2^3 \times 3^2 = 6^5$

24. $3^8 = 9^4$

25. $4^3 + 5^3 = 9^3$

26. $2^3 + 2^5 = 2^3(1 + 2^2)$

27. $\frac{5^{12}}{5^4} = 5^3$

28. **Multiple Choice** Which number is the ones digit of $2^{10} \times 3^{10}$?

A. 2 **B.** 4 **C.** 6 **D.** 8

For Exercises 29 and 30, find the ones digit of the product.

29. $4^{15} \times 3^{15}$

30. $7^{15} \times 4^{20}$

31. Manuela said it must be true that $2^{10} = 2^4 \cdot 2^6$ because she can group $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as $(2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$.

a. Verify that Manuela is correct by evaluating both sides of the equation $2^{10} = 2^4 \cdot 2^6$.

b. Use Manuela's idea of grouping factors to write three other expressions that are equivalent to 2^{10}. Evaluate each expression you find to verify that it is equivalent to 2^{10}.

c. The standard form for 2^7 is 128, and the standard form for 2^5 is 32. Use these facts to evaluate 2^{12}. Explain your work.

d. Test Manuela's idea to see if it works for exponential expressions with other bases, such as 3^8 or $(1.5)^{11}$. Test several cases. Give an argument supporting your conclusion.

Applications

Tell whether the expression is equivalent to 1.25^{10}. Explain your reasoning.

32. $(1.25)^5 \cdot (1.25)^5$

33. $(1.25)^3 \times (1.25)^7$

34. $(1.25) \times 10$

35. $(1.25) + 10$

36. $(1.25^5)^2$

37. $(1.25)^5 \cdot (1.25)^2$

For Exercises 38–41, tell whether the expression is equivalent to $(1.5)^7$. Explain your reasoning.

38. $1.5^5 \times 1.5^2$

39. $1.5^3 \times 1.5^4$

40. 1.5×7

41. $(1.5) + 7$

42. Without actually graphing these equations, describe and compare their graphs. Be as specific as you can.

$y = 4^x \qquad y = 0.25^x \qquad y = 10(4^x) \qquad y = 10(0.25^x)$

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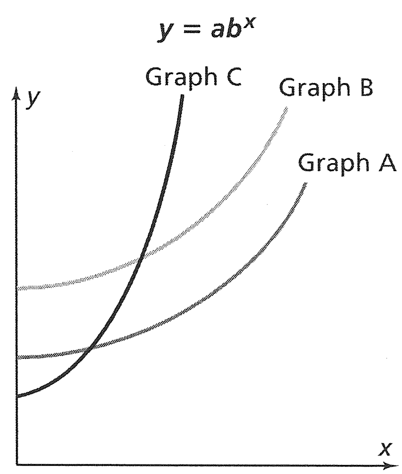
For: Help with Exercise 42
Web Code: ape-3542

43. Each graph below represents an exponential equation of the form $y = ab^x$.

a. For which of the three functions is the value of a greatest?

b. For which of the three functions is the value of b greatest?



Connections

For Exercises 44 and 45, tell whether the statement is *true* or *false*. Do not do an exact calculation. Explain your reasoning.

44. $(1.56892 \times 10^5) - (2.3456 \times 10^4) < 0$

45. $\dfrac{3.96395 \times 10^5}{2.888211 \times 10^7} > 1$

Name ______________________________ Date __________ Class __________

Skill: Simplifying Exponential Expressions

Investigation 5

Growing, Growing, Growing

Find an equivalent expression.

1. $3^2 \cdot 3^5$

2. $1^3 \cdot 1^4$

3. $5^4 \cdot 5^3$

4. $4.5^8 \cdot 4.5^2$

5. $3^3 \cdot 3 \cdot 3^4$

Replace each $\square$ with =, <, or >.

6. $3^8 \square 3 \cdot 3^7$

7. $49 \square 7^2 \cdot 7^2$

8. $5^3 \cdot 5^4 \square 25^2$

Simplify each expression.

9. $\frac{(-3)^6}{(-3)^8}$

10. $\frac{8^4}{8^0}$

11. $\frac{(-4)^8}{(-4)^4}$

12. $\frac{7^5}{7^3}$

13. $\frac{(-3)^5}{(-3)^8}$

Name ______________________ Date ____________ Class ____________

Additional Practice

Investigation 5

Growing, Growing, Growing

1. In parts (a)–(f), write the expression in an equivalent form using exponents. Then write the expression in standard form.

   a. $2^5 \times 2^5$

   b. $4^3 \times 2^5$

   c. 25^4

   d. $\frac{3^4}{3}$

   e. $10^2 \times 2 \times 5$

   f. $3^3 \times 2^3$

2. In parts (a)–(d), find the units digit of the standard form of the expression.

   a. 12^{10}

   b. 11^{23}

   c. 23^{19}

   d. 17^{17}

3. Consider these three equations: $y = 0.625^x$, $y = 0.375^x$, and $y = 1 - 0.5x$.

   a. Sketch graphs of the equations on one set of axes.

   b. What points, if any, do the three graphs have in common?

   c. In which graph does the y-value decrease at a faster and faster rate as the x-value increases?

4. Decide whether each statement is true or false. Explain your reasoning.

   a. $3^5 + 3^5 = 3^{10}$

   b. $5^4 + 2^4 = 7^4$

Name ______________________________ Date __________ Class __________

Skill: Using Exponents

Investigation 1

Growing, Growing, Growing

Write each expression in exponential form.

1. $3 \times 3 \times 3 \times 3 \times 3$

2. $2.7 \times 2.7 \times 2.7$

3. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

4. $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$

Write each expression in standard form.

5. $(0.5)^3$

6. $(2.7)^2$

7. 2^3

8. $(8.1)^3$

Write each number in scientific notation.

9. 480,000

10. 960,000

11. 8,750,000

12. 407,000

Name ______________________________ Date ________________ Period ______

Let's Practice Exponents

1. $\frac{b^5}{b^4}$

2. $4^{10} \cdot 5^{10}$

3. $a^2 \cdot b^4 \cdot a^5$

4. $(8^{-2})^4$

5. $\frac{a^4b^3}{a^2b}$

6. $a^2 \cdot a^4$

7. $\frac{12c^4d^2}{6c^2d}$

8. $a^3 \cdot b^3$

9. $2^{-3} \cdot 2^{-5}$

10. $(a^2b)^3$

11. $m^{-2} \cdot m^5$

12. $(c^4)^5$

13. $4^{-7} \cdot 4^4$

14. $\frac{e^4f^2g^4}{e^3f^5}$

15. $\frac{20ab^3c}{4ab}$

16. $\frac{b^{20}}{b^{20}}$

17. $(2ab^2)^2$

18. $5^3 \cdot 7^2 \cdot 6^2 \cdot 7^4$

19. $b^5 \cdot c^5$

20. $3^5 \cdot 2^5$

21. $\frac{e^6}{e^{-2}}$

22. $(3a^2b^4)^2$

23. $8^{-3} \cdot 8^7$

24. $2^6 \cdot 4^6$

25. $\frac{4^7}{4^1}$

26. $b^{-6} \cdot b^{-11}$

27. $(10^2)^8$

28. $c^{-4} \cdot d^{-4}$

29. $\frac{a^2b^2}{a}$

30. $c^2 \cdot d^3 \cdot c^2 d^4$

31. $(b^2d^2e)^3$

32. $3^4 \cdot 7^4$

33. $a^{10} \cdot b^4 \cdot b^3$

34. $(2de^2)^2$

35. $\dfrac{e^{-6}}{e^{-2}}$

36. $(9^{-2})^2$

37. $10^4 \bullet 10^{-6}$

38. $\dfrac{10^{12}}{10^8}$

39. $(e^2f^4g^3)^2$

40. $d^{10} \bullet e^{10}$

41. $(2d^2e)^3$

42. $\dfrac{14^{-6}}{14^{-8}}$

43. $4^5 \bullet 3^2 \bullet 3^6 \bullet 5^2$

44. $(5^2)^C$

45. $(b^2e)(2ce)$

46. $6^2 \bullet 7^2$

47. $a^8 \bullet a^{-9}$

48. $(.5b^2e^3)^3$

49. $a^4 \bullet b^4$

50. $\dfrac{4^2 \bullet 5^5}{4 \bullet 5^{-2}}$

Name Key Date ____________ Period ______

Let's Practice Exponents

1. $\frac{b^5}{b^4}$ b^1

2. $4^{10} \bullet 5^{10}$ 20^{10}

3. $a^2 \bullet b^4 \bullet a^5$ $a^7 b^4$

4. $(8^{-2})^4$ 8^{-8}

5. $\frac{a^4 b^3}{a^2 b}$ $a^2 b$

6. $a^2 \bullet a^4$ a^6

7. $\frac{12c^4 d^2}{6c^2 d}$ $2c^2 d$

8. $a^3 \bullet b^3$ $(ab)^3$

9. $2^{-3} \bullet 2^{-5}$ 2^{-8}

10. $(a^2 b)^3$ $a^6 b^3$

11. $m^{-2} \bullet m^5$ m^3

12. $(c^4)^5$ c^{20}

13. $4^{-7} \bullet 4^4$ 4^{-3}

14. $\frac{e^4 f^2 g^4}{e^3 f^5}$ $\frac{eg^4}{f^3}$

15. $\frac{20\cancel{a}b^3 c}{4\cancel{a}b}$ $5b^2c$

16. $\frac{b^{20}}{b^{20}}$ b^0 or 1

17. $(2ab^2)^2$ $4a^2b^4$

18. $5^3 \bullet 7^2 \bullet 6^2 \bullet 7^4$ $5^3 \cdot 6^2 \cdot 7^6$

19. $b^5 \bullet c^5$ $(bc)^5$

20. $3^5 \bullet 2^5$ 6^5

21. $\frac{e^6}{e^{-2}}$ e^8

22. $(3a^2b^4)^2$ $9a^4b^8$

23. $8^{-3} \bullet 8^7$ 8^{-21}

24. $2^6 \bullet 4^6$ 8^6

25. $\frac{4^7}{4^1}$ 4^6

26. $b^{-6} \bullet b^{-11}$ b^{-17}

27. $(10^2)^8$ 10^{16}

28. $c^{-4} \bullet d^{-4}$ $(cd)^{-4}$

29. $\frac{a^2 b^2}{a}$ ab^2

30. $c^2 \bullet d^3 \bullet c^2\, d^4$ $c^4 d^7$

31. $(b^2 d^2 e)^3$ $b^6 d^6 e^3$

32. $3^4 \bullet 7^4$ 21^4

33. $a^{10} \bullet b^4 \bullet b^3$ $a^{10} b^7$

34. $(2de^2)^2$ $4d^2e^4$

35. $\frac{e^{-6}}{e^{-2}}$ e^{-4}

36. $(9^{-2})^2$ 9^{-4}

37. $10^4 \cdot 10^{-6}$ 10^{-2}

38. $\frac{10^{12}}{10^8}$ 10^4

39. $(e^2f^4g^3)^2$ $e^4f^8g^6$

40. $d^{10} \cdot e^{10}$ $(de)^{10}$

41. $(2d^2e)^3$ $8d^6e^3$

42. $\frac{14^{-6}}{14^{-8}}$ 14^2

43. $4^5 \cdot 3^2 \cdot 3^6 \cdot 5^2$
$4^5 \cdot 3^8 \cdot 5^2$

44. $(5^2)^C$ 5^{2c}

45. $(b^2e)(2ce)$ $2b^2e^2$

46. $6^2 \cdot 7^2$ 42^2

47. $a^8 \cdot a^{-9}$ a^{-1}

48. $(.5b^2e^3)^3$
$.125b^6e^9$

49. $a^4 \cdot b^4$ $(ab)^4$

50. $\frac{4^2 \bullet 5^5}{4 \bullet 5^{-2}}$ $4 \cdot 5^3$

Name: ______________________________ Date: ______________ Period: _________

Introducing Large Numbers in Scientific Notation

Numbers used in scientific work are often very large.

- For example, there are about 33,400,000,000,000,000,000,000 molecules in 1 gram of water.
- There are about 25,000,000,000,000 red blood cells in a human body.
- According to the 'Big Bang' theory in astronomy, our universe began with an explosion 18,000,000,000 years ago, generating temperatures of 1,000,000,000,000° Celsius.

A calculator is a useful tool for working with large numbers. However, to use your calculator effectively, you need to understand the special way it handles very large and small numbers.

A. Try entering 25,000,000,000,000 on your calculator. Does your calculator allow you to enter all the digits? If you are using a graphing calculator, press $\boxed{\text{ENTER}}$ after you enter the number. What do you think the resulting display means?

B. Use your calculator to find 500,000 x 500,000. What do you think the resulting display means?

The product of 500,000 x 500,000 is 250,000,000,000. However, when you tried to compute this product on your calculator, the display probably showed one of these results.

$\boxed{\text{2.5E11}}$ or $\boxed{\text{2.5} \quad \text{11}}$

Your calculator did not make a mistake. It was using a special notation.

To understand your calculator's notation, let's start by looking at a short way to write 100,000,000,000:

$$\begin{aligned} 100{,}000{,}000{,}000 &= 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \\ &= 10^{11} \end{aligned}$$

In the notation $\mathbf{10^{11}}$ 10 is the ***base*** and 11 is the ***exponent***. The exponent tells you how many times the base is used as a factor. You multiplied 10 by itself eleven times, which is why your exponent is 11.

We can use this short way of writing 100,000,000,000 to find a short way to write 250,000,000,000:

$$250{,}000{,}000{,}000 = 2.5 \times 100{,}000{,}000{,}000$$
$$= 2.5 \times 10^{11}$$

In 2.5×10^{11}, what is the base? ____________ What is the exponent? ____________

- The number 2.5×10^{11} is written in **scientific notation**. A number is written in **scientific notation** if it is expressed in the following form:

A number greater than or equal to 1, but less than 10 **X** **10 raised to an exponent**

- Scientific notation looks a little different on a calculator. Your calculator was using scientific notation when it displayed

  $\boxed{2.5E11}$ or $\boxed{2.5 \quad 11}$

  Both of these displays mean 2.5×10^{11}.

- This example shows how you would use scientific notation to write 4,000,000:

$$4{,}000{,}000 = 4.0 \times 1{,}000{,}000$$
$$= 4.0 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$
$$= 4.0 \times 10^{6}$$

C. How would your calculator display this number?

D. Write each number in standard notation.

i. 10^{22} iii. 10^{11}

ii. 10^{13} iv. 10^{10}

E. Write each number in shorter form by using an exponent.

i. 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10

ii. 1,000,000

F. Write each number in standard notation.

i. 3.0×10^{9}

ii. 2.5×10^{13}

iii. 1.75×10^{10}

G. Write each number in scientific notation.

i. 5,000,000

ii. 18,000,000

iii. 17,900,000,000

H. Look at the calculator displays below. Write each number in standard notation **and** scientific notation.

1. $\boxed{\text{1.7E12}}$ or $\boxed{1.7 \quad 12}$

2. $\boxed{\text{1.7E15}}$ or $\boxed{1.7 \quad 15}$

3. $\boxed{\text{2.35E12}}$ or $\boxed{2.35 \quad 12}$

4. $\boxed{\text{3.368E16}}$ or $\boxed{6.698 \quad 16}$

Name: Key Date: ______________ Period: __________

Introducing Large Numbers in Scientific Notation

Numbers used in scientific work are often very large.

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A calculator is a useful tool for working with large numbers. However, to use your calculator effectively, you need to understand the special way it handles very large and small numbers.

A. Try entering 25,000,000,000,000 on your calculator. Does your calculator allow you to enter all the digits? If you are using a graphing calculator, press $\boxed{\text{ENTER}}$ after you enter the number. What do you think the resulting display means?

answers will vary

B. Use your calculator to find 500,000 x 500,000. What do you think the resulting display means?

2.5×10^{11} calculator displays: 2.5 E11 means

The product of 500,000 x 500,000 is 250,000,000,000. However, when you tried to compute this product on your calculator, the display probably showed one of these results.

$\boxed{\text{2.5E11}}$ or $\boxed{\text{2.5} \quad \text{11}}$

Your calculator did not make a mistake. It was using a special notation.

To understand your calculator's notation, let's start by looking at a short way to write 100,000,000,000:

$$100{,}000{,}000{,}000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$
$$= 10^{11}$$

In the notation $\mathbf{10^{11}}$ 10 is the ***base*** and 11 is the ***exponent***. The exponent tells you how many times the base is used as a factor. You multiplied 10 by itself eleven times, which is why your exponent is 11.

We can use this short way of writing 100,000,000,000 to find a short way to write 250,000,000,000:

$$250{,}000{,}000{,}000 = 2.5 \times 100{,}000{,}000{,}000$$
$$= 2.5 \times 10^{11}$$

In 2.5×10^{11}, what is the base? $\boxed{10}$ What is the exponent? $\boxed{11}$

- The number 2.5×10^{11} is written in **scientific notation**. A number is written in **scientific notation** if it is expressed in the following form:

A number greater than or equal to 1, but less than 10 **X** **10 raised to an exponent**

- Scientific notation looks a little different on a calculator. Your calculator was using scientific notation when it displayed

2.5E11	or	2.5 11

Both of these displays mean 2.5×10^{11}.

- This example shows how you would use scientific notation to write 4,000,000:

$$4{,}000{,}000 = 4.0 \times 1{,}000{,}000$$
$$= 4.0 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$
$$= 4.0 \times 10^{6}$$

C. How would your calculator display this number?

ex: 4E6

D. Write each number in standard notation.

i. 10^{22} 10,000,000,000,000,000,000,000

ii. 10^{13} 10,000,000,000,000

iii. 10^{11} 100,000,000,000

iv. 10^{10} 10,000,000,000

E. Write each number in shorter form by using an exponent.

i. $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ $\boxed{10^{12}}$

ii. 1,000,000 $\boxed{10^{6}}$

F. Write each number in standard notation.

i. 3.0×10^9 3,000,000,000

ii. 2.5×10^{13} 25,000,000,000,000

iii. 1.75×10^{10} 17,500,000,000

G. Write each number in scientific notation.

i. 5,000,000 5×10^6

ii. 18,000,000 1.8×10^7

iii. 17,900,000,000 1.79×10^{10}

H. Look at the calculator displays below. Write each number in standard notation **and** scientific notation.

				standard	scientific
1.	1.7E12	or	1.7 12	1,700,000,000,000	1.7×10^{12}
2.	1.7E15	or	1.7 15	1,700,000,000,000,000	1.7×10^{15}
3.	2.35E12	or	2.35 12	2,350,000,000,000	2.35×10^{12}
4.	3.368E16	or	6.698 16	66,980,000,000,000,000	6.698×10^{16}

NAME______________________________ DATE __________ PERIOD _____

More with Scientific Notation: Large & Small Numbers

Did you know that there are approximately 75,000 genes in each human cell and more than 50 trillion cells in the human body? This means that 75,000 • 50,000,000,000,000 is a low estimate of the number of genes in your body.

Whether you use paper and pencil, an old-fashioned slide rule, or your calculator, exponents are useful when you work with very large numbers. For example, instead of writing 3,750,000,000,000,000,000 genes, scientists write this number more compactly as 3.75×10^{18}. This compact method of writing number is called **scientific notation**. You will learn how to use this notation for large numbers—numbers far from 0 on a number line.

Consider these two lists of numbers.

In scientific notation	**Not in scientific notation**
3.4×10^{5}	27×10^{4}
7.04×10^{3}	120,000,000
6.023×10^{17}	42.682×10^{29}
8×10^{1}	4.2×12^{6}
1.6×10^{2}	$4^{2} \times 10^{2}$

Now you try! Classify each of these numbers as in scientific notation or not. If a number is not in scientific notation, tell why not.

a) 4.7×10^{3}

b) 32×10^{5}

c) $2^{4} \times 10^{6}$

d) 1.107×10^{13}

e) 0.28×10^{11}

Define what it means for a number to be in scientific notation.

____

____

____

Use your calculator's scientific notation mode to help you figure out how to convert standard notation to scientific notation and vice versa.

- Set your calculator to scientific notation mode.
- Enter the number 5000 and press 'Enter.' Your calculator will display its version of 5×10^3. Fill in the top row of the table below with your results.
- Now complete the rest of the rows in the table below.

Standard Notation	**Calculator Notation**	**Scientific Notation**
5000		5×10^3
140,000,000		
-47,000		

Try to complete the next table WITHOUT using a calculator.

Standard Notation	**Calculator Notation**	**Scientific Notation**
7,000,000		
18		
-5,530		
		9.24×10^{10}
		-8.613×10^{12}
		5×10^4
		-1.7×10^5

- In scientific notation, how is the exponent on the 10 related to the number in standard notation?

  ____

- How are the digits before the 10 related to the number in standard notation?

  ____

- If the number in standard notation is negative, how does that show up in scientific notation?

  ____

Write a set of instructions for converting 415,000,000 from standard notation to scientific notation. Assume the person reading your instructions does not have access to a calculator.

____

____

____

Write a set of instructions for converting 6.4×10^5 from scientific notation to standard notation. Assume the person reading your instructions does not have access to a calculator.

____

____

____

You can also use negative exponents to write numbers close to 0 in scientific notation. Just as positive powers of 10 help you rewrite numbers with lots of zeros, negative powers of 10 help you rewrite numbers with lots of zeros between the decimal point and a nonzero digit.

Practice converting from standard notation to scientific notation and vice versa by completing the table below. Use your calculator for the first 4 rows, and then try to do the rest without a calculator.

Standard Notation	Calculator Notation	Scientific Notation
0.0000817		
0.062		
-0.000004905		
0.00000000003		
0.0000012		
-0.000704		
		3.8×10^{-12}
		5×10^{-9}
		-4.107×10^{-5}

Write a set of instructions for converting 0.0000000014 from standard notation to scientific notation. Assume the person reading your instructions does not have access to a calculator.

____

____

____

Write a set of instructions for converting 9.58×10^{-5} from scientific notation to standard notation. Assume the person reading your instructions does not have access to a calculator.

____

____

____

NAME______________________________ **DATE**__________ **PERIOD**_____

More with Scientific Notation: Large & Small Numbers

Now you try!

Convert each number from standard notation to scientific notation.

a) 6,700,000

b) 8,000,000,000

c) -19,000

d) 205,006,000

e) 0.00000099

f) -0.00004

g) 0.0000000105

h) A pi meson, an unstable particle released in a nuclear reaction, "lives" only 0.000000026 second.

Convert each number from scientific notation to standard notation.

a) 9.23×10^{7}

b) 8×10^{4}

c) 4.006×10^{8}

d) -3.1×10^{11}

e) 7.842×10^{-6}

f) -1.03×10^{-5}

g) 5×10^{-10}

h) The mass of an electron is 9.1×10^{-31} kilogram.

NAME Key **DATE** ________ **PERIOD** ____

More with Scientific Notation: Large & Small Numbers

Did you know that there are approximately 75,000 genes in each human cell and more than 50 trillion cells in the human body? This means that 75,000 • 50,000,000,000,000 is a low estimate of the number of genes in your body.

Whether you use paper and pencil, an old-fashioned slide rule, or your calculator, exponents are useful when you work with very large numbers. For example, instead of writing 3,750,000,000,000,000,000 genes, scientists write this number more compactly as 3.75×10^{18}. This compact method of writing number is called **scientific notation**. You will learn how to use this notation for large numbers—numbers far from 0 on a number line.

Consider these two lists of numbers.

In scientific notation	Not in scientific notation
3.4×10^5	27×10^4
7.04×10^3	120,000,000
6.023×10^{17}	42.682×10^{29}
8×10^1	4.2×12^6
1.6×10^2	$4^2 \times 10^2$

Now you try! Classify each of these numbers as in scientific notation or not. If a number is not in scientific notation, tell why not.

a) 4.7×10^3 yes

b) 32×10^5 no because your first number should be between 1 and 10

c) $2^4 \times 10^6$ no because your 1st # should not have an exponent

d) 1.107×10^{13} yes

e) 0.28×10^{11} no because it should be 2.8×10^{10}

Define what it means for a number to be in scientific notation.

The number is factored so that the first factor is between 1 + 10 and the second factor is a power of 10.

Use your calculator's scientific notation mode to help you figure out how to convert standard notation to scientific notation and vice versa.

- Set your calculator to scientific notation mode.
- Enter the number 5000 and press 'Enter.' Your calculator will display its version of 5 x 10^3. Fill in the top row of the table below with your results.
- Now complete the rest of the rows in the table below.

Standard Notation	Calculator Notation	Scientific Notation
5000	5 E^3	5 x 10^3
140,000,000	1.4 E^8	1.4 x 10^8
-47,000	-4.7E^4	-4.7 x 10^4

Try to complete the next table WITHOUT using a calculator.

Standard Notation	Calculator Notation	Scientific Notation
7,000,000	7E^6	7x10^6
18	1.8E^1	1.8 x 10^1
-5,530	-5.53E^3	5.53 x 10^3
92,400,000,000	9.24E^{10}	9.24 x 10^{10}
-8,613,000,000,000	-8.613E^{12}	-8.613 x 10^{12}
50,000	5E^4	5 x 10^4
-170,000	-1.7E^5	-1.7 x 10^5

- In scientific notation, how is the exponent on the 10 related to the number in standard notation?
  Its the # of digits following the first digit in the original number
- How are the digits before the 10 related to the number in standard notation?
  They show the significant digits in the original number.

- If the number in standard notation is negative, how does that show up in scientific notation?

The 1st factor in scientific notation is also negative.

Write a set of instructions for converting 415,000,000 from standard notation to scientific notation. Assume the person reading your instructions does not have access to a calculator.

① Write 415 w/ one digit before decimal (4.15). ② Determine how many places the decimal point needs to move to have 4.15 become 415,000,000. ③ make this the exponent on 10. ④ you get: 4.15×10^8.

Write a set of instructions for converting 6.4×10^5 from scientific notation to standard notation. Assume the person reading your instructions does not have access to a calculator.

move the decimal point in 6.4 five places to the right as shown in 10^5. The standard notation is: 640,000.

You can also use negative exponents to write numbers close to 0 in scientific notation. Just as positive powers of 10 help you rewrite numbers with lots of zeros, negative powers of 10 help you rewrite numbers with lots of zeros between the decimal point and a nonzero digit.

Practice converting from standard notation to scientific notation and vice versa by completing the table below. Use your calculator for the first 4 rows, and then try to do the rest without a calculator.

Standard Notation	Calculator Notation	Scientific Notation
0.0000817	8.17E-5	8.17×10^{-5}
0.062	6.2E-2	6.2×10^{-2}
-0.000004905	-4.905E-6	-4.905×10^{-6}
0.00000000003	3E-11	3×10^{-11}
0.0000012	1.2E-6	1.2×10^{-6}
-0.000704	-7.04E-4	-7.04×10^{-4}
.0000000000038	3.8E-12	3.8×10^{-12}
.000000005	5E-9	5×10^{-9}
.00004107	4.107E-5	-4.107×10^{-5}

Write a set of instructions for converting 0.0000000014 from standard notation to scientific notation. Assume the person reading your instructions does not have access to a calculator.

① 14 with w/ digit before the decimal (1.4). ② determine how many place values the decimal point needs to move to have .0000000014 become 1.4. ③ make this # negative and it becomes the exponent on 10.

Write a set of instructions for converting 9.58×10^{-5} from scientific notation to standard notation. Assume the person reading your instructions does not have access to a calculator.

① take the decimal out of 9.58 (958). ② move the decimal point 5 places to the left ③ your answer is .0000958.

NAME_______________________________ DATE __________ PERIOD _____

More with Scientific Notation: Large & Small Numbers

Now you try!

Convert each number from standard notation to scientific notation.

a) 6,700,000 6.7×10^6

b) 8,000,000,000 8×10^9

c) -19,000 -1.9×10^4

d) 205,006,000 2.05006×10^8

e) 0.00000099 9.9×10^{-7}

f) -0.00004 -4×10^{-5}

g) 0.0000000105 1.05×10^{-8}

h) A pi meson, an unstable particle released in a nuclear reaction, "lives" only 0.000000026 second.

2.6×10^{-8}

Convert each number from scientific notation to standard notation.

a) 9.23×10^7 92300000

b) 8×10^4 80000

c) 4.006×10^8 400600000

d) -3.1×10^{11} -310000000000

e) 7.842×10^{-6} .000007842

f) -1.03×10^{-5} .0000103

g) 5×10^{-10} .0000000005

h) The mass of an electron is 9.1×10^{-31} kilogram.

.00000000000000000000000000000091

Name ______________________________ Date ____________________ Period ______

Scientific Notation: Adding and Subtracting

Use your knowledge of exponents and scientific notation to find a pattern in the problems below.

A. 1. Expand each of the following to explain why each of the following statements is true.

a. $2 \times 10^3 + 4 \times 10^3 = 6 \times 10^3$

b. $3.1 \times 10^4 + 2.7 \times 10^4 = 5.8 \times 10^4$

c. $5 \times 10^5 + 6 \times 10^5 = 11 \times 10^5 = 1.1 \times 10^6$

2. Give another example that fits the pattern in part (1).

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$a \times 10^m + b \times 10^m =$ __________

B. 1. Expand each of the following to explain why each of the following statements is true.

a. $5 \times 10^3 - 3 \times 10^3 = 2 \times 10^3$

b. $6.8 \times 10^4 - 4.7 \times 10^4 = 2.1 \times 10^4$

c. $14 \times 10^2 - 3 \times 10^2 = 11 \times 10^2 = 1.1 \times 10^3$

2. Give another example that fits the pattern in part (1).

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$a \times 10^m - b \times 10^m =$ ____________

C. Look at the addition and subtraction problems below.

$$3.76 \times 10^4 + 5.5 \times 10^2$$

$$2.19 \times 10^3 - 3.947 \times 10^2$$

$$7.3 \times 10^3 + 2.1 \times 10^4$$

1. Compare these addition problems to the ones you looked at earlier in problems A1 and B1. What makes these addition problems different than the ones you already studied?

2. In the problems we looked at in A1 and B1, you were able to see a pattern for how to add and subtract problems in scientific notation.

   Look at the addition problem below. How do you "change" this problem so that you can follow the pattern previously discussed and subtract these two large numbers?

$$2.19 \times 10^3 - 3.947 \times 10^2$$

3. Why do the exponents have to be the same to add or subtract numbers written in scientific notation?

4. What will you have to do first to solve the problem below? Be specific.

$$3.76 \times 10^4 + 5.5 \times 10^2$$

5. Solve.

a. $3.76 \times 10^4 + 5.5 \times 10^2$

b. $5.4681 \times 10^4 - 9.1472 \times 10^3$

6. The distance from Earth to Mars is 5.6×10^6. The distance from Mars to the sun is 2.49×10^8. What is the total distance from Earth to the Sun?

7. An ant climbs 8 feet in 1.2×10^{-3} mph. A worm climbs the same distance in 9.8×10^{-4} mph. How much faster is the ant than the worm?

8. $1.8 \times 10^1 + 7.7 \times 10^2$

9. $8 \times 10^2 + 5.5 \times 10^0$

10. Eyelash mites live on your eyelashes for 2.3×10^2 minutes. Bed bugs live on your pillow for 3.62×10^3 minutes. How much longer do the bed bugs live than the bed bugs?

11. 2.6 million years ago the first cockroach roamed the Earth. 5×10^{-1} years ago your teacher found one roaming around your classroom. How much time passed between the time your teacher found the cockroach in the classroom and the first one roaming the Earth?

Name ______________________________ Date ____________________ Period ______

Scientific Notation: Adding and Subtracting

Use your knowledge of exponents and scientific notation to find a pattern in the problems below.

A. 1. Expand each of the following to explain why each of the following statements is true.

a. $2 \times 10^3 + 4 \times 10^3 = 6 \times 10^3$

2000 + 4000 = 6000

6000 = 6×10^3

b. $3.1 \times 10^4 + 2.7 \times 10^4 = 5.8 \times 10^4$

31000 + 27000 = 58000

58000 = 5.8×10^4

c. $5 \times 10^5 + 6 \times 10^5 = 11 \times 10^5 = 1.1 \times 10^6$

500000 + 600000 = 110000

110000 = 1.1×10^6

2. Give another example that fits the pattern in part (1).

answers will vary

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$a \times 10^m + b \times 10^m =$ $(a+b) \times 10^m$

B. 1. Expand each of the following to explain why each of the following statements is true.

a. $5 \times 10^3 - 3 \times 10^3 = 2 \times 10^3$

5000 - 3000 = 2000 2000 = 2×10^3

b. $6.8 \times 10^4 - 4.7 \times 10^4 = 2.1 \times 10^4$

68000 - 47000 = 21000 21000 = 2.1×10^4

c. $14 \times 10^2 - 3 \times 10^2 = 11 \times 10^2 = 1.1 \times 10^3$

1400 - 300 = 1100 1100 = 1.1×10^3

(11×10^2)

2. Give another example that fits the pattern in part (1).

answers will vary

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$a \times 10^m - b \times 10^m =$ $(a-b) \times 10^m$

C. Look at the addition and subtraction problems below.

$$3.76 \times 10^4 + 5.5 \times 10^2$$

$$2.19 \times 10^3 - 3.947 \times 10^2$$

$$7.3 \times 10^3 + 2.1 \times 10^4$$

1. Compare these addition problems to the ones you looked at earlier in problems A1 and B1. What makes these addition problems different than the ones you already studied?

The exponents attached to the tens are different in each problem.

2. In the problems we looked at in A1 and B1, you were able to see a pattern for how to add and subtract problems in scientific notation.

Look at the addition problem below. How do you "change" this problem so that you can follow the pattern previously discussed and subtract these two large numbers?

$$2.19 \times 10^3 - 3.947 \times 10^2$$

Either change 2.19×10^3 to 21.9×10^2

- or -

3.947×10^2 to $.3947 \times 10^3$

3. Why do the exponents have to be the same to add or subtract numbers written in scientific notation?

when adding numbers with decimals, the place values must line up. For this to happen you must be multiplying the first numbers in the scientific notation by the same power of ten before you can add or subtract them.

4. What will you have to do first to solve the problem below? Be specific.

$$3.76 \times 10^4 + 5.5 \times 10^2$$

change 5.5×10^2 to $.055 \times 10^4$

- or -

change 3.76×10^4 to 376×10^2

5. Solve.

a. $3.76 \times 10^4 + 5.5 \times 10^2$

$376 \times 10^2 + 5.5 \times 10^2$

$381.5 \times 10^2 = \boxed{3.815 \times 10^4}$

b. $5.4681 \times 10^4 - 9.1472 \times 10^3$

$54.681 \times 10^3 - 9.1472 \times 10^3$

$45.5338 \times 10^3 = 4.55338 \times 10^4$

6. The distance from Earth to Mars is 5.6×10^6. The distance from Mars to the sun is 2.49×10^8. What is the total distance from Earth to the Sun?

$5.6 \times 10^6 + 2.49 \times 10^8$

$.056 \times 10^8 + 2.49 \times 10^8$

$\boxed{2.546 \times 10^8}$

$\boxed{\text{or}}$

5600000 +
249000000 =
254600000
2.546×10^8

7. An ant climbs 8 feet in 1.2×10^{-3} mph. A worm climbs the same distance in 9.8×10^{-4} mph. How much faster is the ant than the worm?

worm .00098

ant .0012

.0012 - .00098 = .00022

$\boxed{2.2 \times 10^{-4}}$

8. $1.827 \times 10^1 + 7.747 \times 10^2$

$1.8 \times 10^1 + 7.7 \times 10^2$

$.18 \times 10^2 + 7.7 \times 10^2$

$\boxed{7.88 \times 10^2}$

9. $8.0055 \times 10^2 + 5.5601 \times 10^0$

$8 \times 10^2 + 5.5 \times 10^0$

$8 \times 10^2 + .055 \times 10^2$

$\boxed{8.055 \times 10^2}$

10. ~~$5.4681 \times 10^4 - 9.1472 \times 10^3$~~

11. ~~$6.87 \times 10^4 - 7.5 \times 10^3$~~

10 ~~12~~. Eyelash mites live on your eyelashes for 2.3×10^2 minutes. Bed bugs live on your pillow for 3.62×10^3 minutes. How much longer do the bed bugs live than the bed bugs?

$3.62 \times 10^3 - 2.3 \times 10^2$

$36.2 \times 10^2 - 2.3 \times 10^2$

33.9×10^2

3.39×10^3

3620 - 230 =

3390 =

3.39×10^3

11 ~~13~~. 2.6 million years ago the first cockroach roamed the Earth. 5×10^{-1} years ago your teacher found one roaming around your classroom. How much time passed between the time your teacher found the cockroach in the classroom and the first one roaming the Earth?

2600000 - .5 = 2599999.5

$\boxed{2.5999995 \times 10^6}$

years

Name ________________________________ Date ____________________ Period ______

Scientific Notation: Multiplication and Division

Many of the numbers occurring in science are either very large or very small. The speed of light is 983,569,000 feet per second. One millimeter is equal to 0.000001 kilometers. In scientific notation, numbers larger than 10 or smaller than 1 are written by using positive or negative exponents.

An important feature of scientific notation is its use in computations. Numbers in scientific notation are nothing more than exponential expressions, and you have already studied operations with exponential expressions. We use the same rules of exponents on numbers in scientific notation that we use on any other exponential expressions.

Use your knowledge of exponents and scientific notation to find a pattern in the problems below.

A. 1. Expand each of the following to explain why each of the following statements are true.

a. $2 \times 10^2 \bullet 4 \times 10^3 = 8 \times 10^5$

b. $3 \times 10^4 \bullet 9 \times 10^6 = 27 \times 10^{10}$

c. $5 \times 10^5 \bullet 3 \times 10^2 = 15 \times 10^7$

2. Give another example that fits the pattern in part (1).

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$a \times 10^m \bullet b \times 10^n =$ _______________

B. 1. Expand each of the following to explain why each of the following statements is true

a. $\dfrac{6x10^4}{2x10^2} = 3 \times 10^2$

b. $\dfrac{9x10^6}{3x10^3} = 3 \times 10^3$

c. $\dfrac{8x10^7}{4x10^3} = 2 \times 10^4$

2. Give another example that fits the pattern in part (1).

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$$\frac{(a)x10^{m}}{(b)x10^{n}} = \underline{\hspace{4cm}}$$

Practice:

1. $6 \times 10^{6} \bullet 4 \times 10^{5}$

2. $8 \times 10^{-3} \bullet 7 \times 10^{-3}$

3. $1.5 \times 10^{3} \bullet 2 \times 10^{4}$

4. $6 \times 10^{5} \bullet 7 \times 10^{2}$

5. $9 \times 10^{-6} \bullet 3 \times 10^{3}$

6. $5 \times 10^{8} \bullet 6 \times 10^{4}$

7. $8 \times 10^{9} \bullet 3 \times 10^{4}$

8. $3 \times 10^{-2} \times 6 \times 10^{-5}$

9. $\dfrac{4x10^{6}}{2x10^{5}}$

10. $\dfrac{5x10^{7}}{2x10^{8}}$

11. $\dfrac{6x10^{5}}{3x10^{3}}$

12. $\dfrac{1.2x10^{5}}{4x10^{8}}$

13. $\dfrac{1.5x10^{8}}{3x10^{3}}$

14. $\dfrac{4.8x10^{9}}{6x10^{9}}$

15. $\dfrac{3x10^{-3}}{1x10^{6}}$

16. $\dfrac{2.4x10^{2}}{8x10^{7}}$

17. Americans make almost 2 billion telephone calls each day. (www.britannica.com)

a. Write this number in standard notation and in scientific notation.

b. How many phone calls do Americans make in one year? (Assume that there are 365 days in a year.) Write your answer in scientific notation

18. The speed of light is 3×10^{8} meters/second. If the sun is 1.5×10^{11} meters from earth, how many seconds does it take light to reach the earth. Express your answer in scientific notation

20. How many times bigger is the sun (1.3×10^{9}) to a giant squid (2×10^{2})?

21. Light travels at approximately 3.0×10^{8} m/sec. How far does light travel in one week?

22. Approximately $.9 \times 10^{5}$ gun crimes are reported to police every day. How many gun crimes are reported every hour?

23. Assume that there are 20,000 runners in the New York City Marathon. Each runner runs a distance of 26 miles. If you add together the total number of miles for all runners, how many times around the globe would the marathon runners have gone?
Consider the circumference of the earth to be 2.5×10^{4} miles.

Name ______________________ Date ______________ Period _____

Scientific Notation: Multiplication and Division

Many of the numbers occurring in science are either very large or very small. The speed of light is 983,569,000 feet per second. One millimeter is equal to 0.000001 kilometers. In scientific notation, numbers larger than 10 or smaller than 1 are written by using positive or negative exponents.

An important feature of scientific notation is its use in computations. Numbers in scientific notation are nothing more than exponential expressions, and you have already studied operations with exponential expressions. We use the same rules of exponents on numbers in scientific notation that we use on any other exponential expressions.

Use your knowledge of exponents and scientific notation to find a pattern in the problems below.

A. 1. Expand each of the following to explain why each of the following statements are true.

a. $2 \times 10^2 \bullet 4 \times 10^3 = 8 \times 10^5$

200 · 4000 = 800,000 800,000 = 8×10^5

b. $3 \times 10^4 \bullet 9 \times 10^6 = 27 \times 10^{10}$

30000 · 9000000 = 270,000,000,000 270000000000 = 27×10^{10}

c. $5 \times 10^5 \bullet 3 \times 10^2 = 15 \times 10^7$

500000 · 300 = 150000000 = 15×10^7

2. Give another example that fits the pattern in part (1).

answers will vary

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$a \times 10^m \bullet b \times 10^n = (a \cdot b) \times 10^{(m+n)}$

B. 1. Expand each of the following to explain why each of the following statements is true

a. $\frac{6x10^4}{2x10^2} = 3 \times 10^2$ $\frac{60000}{200}$ = 300 OR 3×10^2

b. $\frac{9x10^6}{3x10^3} = 3 \times 10^3$ $\frac{9000000}{3000}$ = 3000 OR 3×10^3

c. $\frac{8x10^7}{4x10^3} = 2 \times 10^4$ $\frac{80000000}{4000}$ = 20000 OR 2×10^4

2. Give another example that fits the pattern in part (1).

answers will vary

3. Complete the following equation to show how you can find the base and exponent of the product when you multiply two powers with the same exponent. Explain your reasoning.

$$\frac{(a)x10^m}{(b)x10^n} = (a \div b) \times 10^{(m-n)}$$

Practice:

1. $6 \times 10^6 \bullet 4 \times 10^5$ 24×10^{11} $\boxed{2.4 \times 10^{12}}$

2. $8 \times 10^{-3} \bullet 7 \times 10^{-3}$ 56×10^{-6} $\boxed{5.6 \times 10^{-5}}$

3. $1.5 \times 10^3 \bullet 2 \times 10^4$ 3×10^7

4. $6 \times 10^5 \bullet 7 \times 10^2$ 42×10^7 $\boxed{4.2 \times 10^8}$

5. $9 \times 10^{-6} \bullet 3 \times 10^3$ 27×10^{-3} $\boxed{2.7 \times 10^{-2}}$

6. $5 \times 10^8 \bullet 6 \times 10^4$ $\cancel{30 \times 10^{12}}$ $\boxed{3 \times 10^{13}}$

7. $8 \times 10^9 \bullet 3 \times 10^4$ $\boxed{24 \times 10^{13}}$

8. $3 \times 10^{-2} \times 6 \times 10^{-5}$ $\cancel{18 \times 10^{-3}}$ $\boxed{1.8 \times 10^{-2}}$

9. $\frac{4x10^6}{2x10^5}$ $\boxed{2 \times 10^1}$

10. $\frac{5x10^7}{2x10^8}$ $\boxed{2.5 \times 10^{-1}}$

11. $\frac{6x10^5}{3x10^3}$ $\boxed{2 \times 10^2}$

12. $\frac{1.2x10^5}{4x10^8}$ $.3 \times 10^{-3}$ $\boxed{3 \times 10^{-4}}$

13. $\frac{1.5x10^8}{3x10^3}$ $.5 \times 10^5$ $\boxed{5 \times 10^4}$

14. $\frac{4.8x10^9}{6x10^9}$ $.8 \times 10^0$ $\boxed{8 \times 10^{-1}}$

15. $\frac{3x10^{-3}}{1x10^6}$ $\boxed{3 \times 10^{-9}}$

16. $\frac{2.4x10^2}{8x10^7}$ $.3 \times 10^{-5}$ $\boxed{4 \times 10^{-6}}$

17. Americans make almost 2 billion telephone calls each day. (www.britannica.com)

a. Write this number in standard notation and in scientific notation.

2000000000 -or- ~~2.0 x 10~~

2×10^9

b. How many phone calls do Americans make in one year? (Assume that there are 365 days in a year.) Write your answer in scientific notation

$2 \times 10^9 \times 365 \times 10^0$

730×10^9 7.30×10^{11}

18. The speed of light is 3 x 10 ^{8} meters/second. If the sun is 1.5x 10^{11} meters from earth, how many seconds does it take light to reach the earth. Express your answer in scientific notation

$1.5 \times 10^{11} \times 3 \times 10^8$

$\boxed{4.5 \times 10^{19}}$

20. How many times bigger is the sun (1.3 x 10^9) to a giant squid (2 x 10^2)?

$\frac{1.3 \times 10^9}{2 \times 10^2} = .65 \times 10^7$

$\boxed{6.5 \times 10^6}$

21. Light travels at approximately 3.0 x 10^8 m/sec. How far does light travel in one ~~week~~ hour?

$3 \times 10^8 \times 60 = 1.8 \times 10^{10}$

18,000000000 x 60

$\boxed{1.08 \times 10^{12}}$

22. Approximately .9 x 10^5 gun crimes are reported to police every day. How many gun crimes are reported every hour?

$\frac{.9 \times 10^5}{24 \times 10^0}$ $.0375 \times 10^5$

$\boxed{3.75 \times 10^3}$

23. Assume that there are 20,000 runners in the New York City Marathon. Each runner runs a distance of 26 miles. If you add together the total number of miles for all runners, how many times around the globe would the marathon runners have gone?
Consider the circumference of the earth to be 2.5 x 10^4 miles.

~~20000 x 26~~

~~520000~~

~~$\frac{2.5 \times 10^4}{5.2 \times 10^5}$~~

Name: ______________________________ Date: _____________ Per: _____

Scientific Notation: Operations Practice

Solve each of the problems below. Be sure to show all your work.

1. There are approximately 5.58×10^{21} atoms in a gram of silver. How many atoms are there in 3 kilograms of silver? (1000 grams = 1 kilogram). Express your answer in scientific notation.

2. Because the number of molecules in a given amount of a compound is usually a very large number, scientists often work with a quantity called a mole. One mole is about 6.02×10^{23} molecules. How many molecules are in 3×10^{5} moles?

3. Cal and Al were assigned this multiplication problem for homework:

$$(3.5 \times 10^{4})\ (14.8 \times 10^{5})$$

Cal got an answer of 51.8×10^{9}, and Al got 5.18×10^{10}.

a. Are Cal's and Al's answers equivalent? Explain why or why not.

b. Whose answers is in scientific notation? How do you know?

c. Find another exponential expression equivalent to Cal's and Al's answers.

4. Explain how you can rewrite a number, such as 432.5×10^3, in scientific notation.

5. A lake in Minnesota covers an area of about 8.5×10^7 square feet and its average depth is about 32 feet. How much water does the lake hold?

6. Teens use slang words over 1.1 million times by age 15.

a. Write this number in standard notation and in scientific notation.

b. How many slang words do Americans teens say in a year? Write your answer in scientific notation.

7. On average a person sheds 1 million dead skills cells every 40 minutes.

a. How many dead skin cells does a person shed in an hour? Write your answer in scientific notation.

b. How many dead skin cells does a person shed in a year? (Assume there are 365 days in a year). Write your answer in scientific notation.

8. A light-year is the distance light can travel in one year. This distance is approximately 9460 billion kilometers. The Milky Way is estimated to be about 100,000 light years in diameter.

a. Write both distances in scientific notation.

b. Find the diameter of the Milky Way in kilometers. Use Scientific Notation.

c. Scientists estimate the diameter of the earth is greater than 1.27×10^4 km. How many times larger is the diameter of the Milky Way?

Name: ______________________________ Date: _____________ Per: _____

Scientific Notation: Operations Practice

Solve each of the problems below. Be sure to show all your work.

1. There are approximately 5.58×10^{21} atoms in a gram of silver. How many atoms are there in 3 kilograms of silver? (1000 grams = 1 kilogram). Express your answer in scientific notation.

$5.58 \times 10^{21} \times 1000 =$

5580×10^{21} atoms in 1 kg.

$5580 \times 10^{21} \times 3$ kg $= 16740 \times 10^{21}$

or

$\boxed{1.6740 \times 10^{25}}$

2. Because the number of molecules in a given amount of a compound is usually a very large number, scientists often work with a quantity called a mole. One mole is about 6.02×10^{23} molecules. How many molecules are in 3×10^5 moles?

$6.02 \times 10^{23} \times 3 \times 10^5$

18.06×10^{28}

$\boxed{1.806 \times 10^{29}}$

3. Cal and Al were assigned this multiplication problem for homework:

$$(3.5 \times 10^4)\ (14.8 \times 10^5)$$

Cal got an answer of 51.8×10^9, and Al got 5.18×10^{10}.

a. Are Cal's and Al's answers equivalent? Explain why or why not.

yes they are. to get 51.8 from 5.18 you need to multiply 5.18 by one more power of 10 which is why the problem says 5.18×10^{10} instead of 10^9.

b. Whose answers is in scientific notation? How do you know?

al's! He multiplied a number between 1 & 10 by a power of 10.

c. Find another exponential expression equivalent to Cal's and Al's answers.

ex: $.518 \times 10^{11}$

4. Explain how you can rewrite a number, such as 432.5×10^3, in scientific notation.

① move the decimal between the 4 & 3 (4.325)

② you changed 432.5 to a smaller number. In fact its 100 times smaller. so you need to multiply 4.325 by 2 more powers of 10 so 432.5×10^3 becomes

$\boxed{4.325 \times 10^5}$

5. A lake in Minnesota covers an area of about 8.5×10^7 square feet and its average depth is about 32 feet. How much water does the lake hold?

$8.5 \times 10^7 \times 32 \times 10^0$

$272 \times 10^7 =$

$\boxed{2.72 \times 10^9}$ cubic feet

6. Teens use slang words over 1.1 million times by age 15.

a. Write this number in standard notation and in scientific notation.

1.1 million = 1,100,000 standard

1.1×10^6 scientific

b. How many slang words do Americans teens say in a year? Write your answer in scientific notation.

$1100000 \times 15 = 16500000$

$\boxed{1.65 \times 10^7}$

7. On average a person sheds 1 million dead skills cells every 40 minutes.

a. How many dead skin cells does a person shed in an hour? Write your answer in scientific notation.

$$\frac{1\,000\,000 \times 1.5}{40 \text{ min.} \times 1.5} \quad \frac{\boxed{1{,}500{,}000}}{60 \text{ min}} \rightarrow \boxed{1.5 \times 10^6}$$

b. How many dead skin cells does a person shed in a year? (Assume there are 365 days in a year). Write your answer in scientific notation.

$1.5 \times 10^6 \times 24 = 36 \times 10^6$

$36 \times 10^6 \times 365 = 13140 \times 10^6$

$\boxed{1.3140 \times 10^{10}}$

8. A light-year is the distance light can travel in one year. This distance is approximately 9460 billion kilometers. The Milky Way is estimated to be about 100,000 light years in diameter.

a. Write both distances in scientific notation.

$9{,}460{,}000{,}000{,}000 = \boxed{9.46 \times 10^{12}}$

$100{,}000 = \boxed{1 \times 10^5}$

b. Find the diameter of the Milky Way in kilometers. Use Scientific Notation.

$9.46 \times 10^{12} \times 1 \times 10^5$

$\boxed{9.46 \times 10^{17}}$

c. Scientists estimate the diameter of the earth is greater than 1.27 x 104km. How many times larger is the diameter of the Milky Way?

10^4

1.27×10^4 km

$$\frac{9.46 \times 10^{17}}{1.27 \times 10^4}$$

$\boxed{7.44 \times 10^{13}}$

Say It with Symbols

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	<u>**6-8 Performance Expectations**</u>
1.1 Adding & Multiplying, p. 6	2		Must do	**8.1.A Solve one-variable linear equations.**
1.2 Dividing, p. 8	1		Should do	
1.3 Working with Exponents, p. 10	1		Should do	Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Inv. 1 Reflections	1			
2.1 Tiling Pools, p. 20	1		Should do	
2.2 Thinking in Different Ways, p. 22	1		Must do	
2.3 Diving In, p. 23	1		Must do	
Inv. 2 Reflections, p. 33	1			
3.1 Walking Together, p. 36 (vocab. start of Inv 3 important)	1		Must do	
3.2 Estimating Profit p. 38	1		Must do	
Quiz	1			
4.1 Comparing Cost, p. 53	1		Must do	
4.2 Solving Linear Equations, p. 54	2		Must do	
4.3 Reasoning with Symbols, p. 56	2		Must do	
(CMP2) Say it With Symbols ACE Questions pgs. 31, 32, 46, 47	1	Binder or CMP2 disk		
Unit Assessment	1			
Total Instructional:	**19 days**			

Contents in Say It with Symbols

Say It w/ Symbols

12. The math club is selling posters to advertise National Algebra day. The following equation represents the profits P they expect for selling n posters at x dollars.

$$P = xn - 6n$$

They also know that the number of posters n sold depends on the selling price x, which is represented by this equation:

$$n = 20 - x$$

a. Write an equation for profit in terms of the number of posters sold. **Hint:** First solve the equation $n = 20 - x$ for x.

b. What is the profit for selling 10 posters?

c. What is the selling price of the posters in part (b)?

d. What is the greatest possible profit?

Connections

13. **Multiple Choice** Which statement is *false* when a, b, and c are different real numbers?

F. $(a + b) + c = a + (b + c)$ **G.** $ab = ba$

H. $(ab)c = a(bc)$ **J.** $a - b = b - a$

For Exercises 14–16, use the Distributive Property and sketch a rectangle to show the equivalence.

14. $x(x + 5)$ and $x^2 + 5x$

15. $(2 + x)(2 + 3x)$ and $4 + 8x + 3x^2$

16. $(x + 2)(2x + 3)$ and $2x^2 + 7x + 6$

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Web Code: apa-6254

17. Some steps are missing in the solution to $11x - 12 = 30 + 5x$.

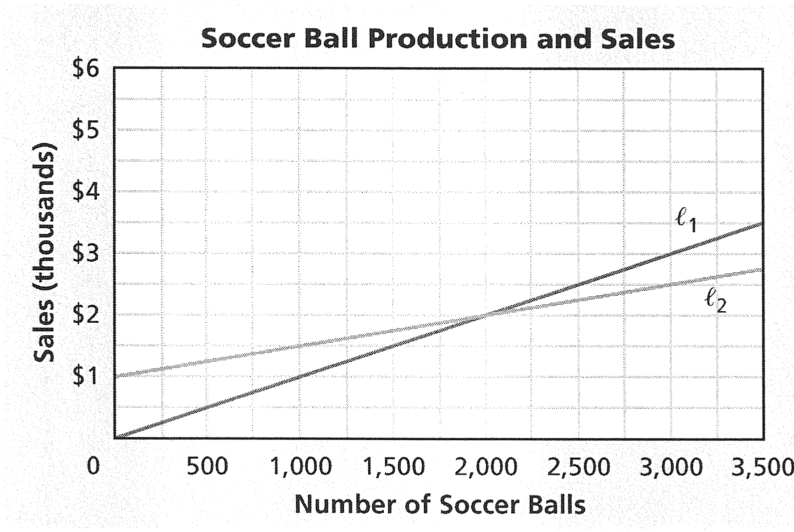
$$11x - 12 = 30 + 5x$$
$$11x = 42 + 5x$$
$$6x = 42$$
$$x = 7$$

a. Copy the steps above. Fill in the missing steps.

b. How can you check that $x = 7$ is the correct solution?

c. Explain how you could use a graph or a table to solve the original equation for x.

18. In the following graph, line ℓ_1 represents the income for selling n soccer balls. Line ℓ_2 represents the expenses of manufacturing n soccer balls.



a. What is the start-up expense (the expense before any soccer balls are produced) for manufacturing the soccer balls? NOTE: The vertical axis is in *thousands* of dollars.

b. What are the expenses and income for producing and selling 500 balls? For 1,000 balls? For 3,000 balls? Explain.

c. What is the profit for producing and selling 500 balls? For 1,000 balls? For 3,000 balls? Explain.

d. What is the break-even point? Give the number of soccer balls and the expenses.

e. Write equations for the expenses, income, and profit. Explain what the numbers and variables in each equation represent.

f. Suppose the manufacturer produces and sells 1,750 soccer balls. Use the equations in part (e) to find the profit.

g. Suppose the profit is $10,000. Use the equations in part (e) to find the number of soccer balls produced and sold.

For Exercises 19–24, use properties of equality to solve the equation. Check your solution.

19. $7x + 15 = 12x + 5$

20. $7x + 15 = 5 + 12x$

21. $-3x + 5 = 2x - 10$

22. $14 - 3x = 1.5x + 5$

23. $9 - 4x = \frac{3 + x}{2}$

24. $-3(x + 5) = \frac{2x - 10}{3}$

SWS - ACE answers pg. 31 & 32

12. **a.** First we need to write the equation $n = 20 - x$ in terms of $x = 20 - n$. So substituting into $P = xn - 6n$ we get that $P = (20 - n)n - 6n$ or $P = 20n - n^2 - 6n$ which is equivalent to $P = 14n - n^2$.

b. \$40; using the equation $P = 14n - n^2$, substitute 10 in for n. The profit is $P = 14(10) - 10^2 = 40$.

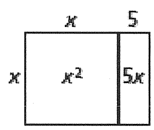
c. The selling price can be found using the equation $n = 20 - x$. So when $n = 10$ the selling price is \$10.

d. \$49; the greatest profit can be found by making a table or graph for the profit equation $P = 20n - n^2 - 6n = 14n - n^2$. The greatest profit occurs when they sell 7 posters, which yields a value of $P = 14(7) - 7^2 = 49$.

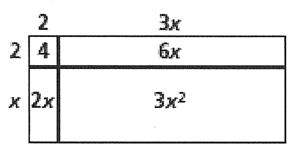
Connections

13. J; Students can try an example like $a = 1$ and $b = 2$ to check that J is false. The other letters are true: F and H are the Associative Property of Addition and Multiplication, respectively, and G is the Commutative Property of Multiplication.

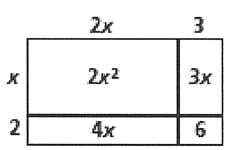
14. $x(x + 5) = x^2 + 5x$



15. $(2 + x)(2 + 3x) = (2 + x)2 + (2 + x)3x = 4 + 2x + 6x + 3x^2 = 4 + 8x + 3x^2$



16. $(x + 2)(2x + 3) = (x + 2)2x + (x + 2)3 = 2x^2 + 4x + 3x + 6 = 2x^2 + 7x + 6$



17. a.

$$\begin{aligned} 11x - 12 &= 30 + 5x \\ 11x - 12 + 12 &= 30 + 12 + 5x \\ 11x &= 42 + 5x \\ 11x - 5x &= 42 + 5x - 5x \\ 6x &= 42 \\ \frac{6x}{6} &= \frac{42}{6} \\ x &= 7 \end{aligned}$$

b. To check, substitute 7 into the original equation for x and see if the values on each side of the equal sign are equal to each other.

$$\begin{aligned} 11x - 12 &= 30 + 5x \\ 11(7) - 12 &= 30 + 5(7) \\ 77 - 12 &= 30 + 35 \\ 65 &= 65 \end{aligned}$$

c. To solve the equation using a graph, first graph each of the equations $y = 11x - 12$ and $y = 30 + 5x$ and use the x-value of their point of intersection for the solution. To solve the equation using a table, look on the tables for each equation and see for which value of x their y-values coincide.

18. a. \$1,000

b.

Number of Soccer Balls	Income from Soccer Balls	Expenses of Soccer Balls
500	500	1,250
1,000	1,000	1,500
3,000	3,000	2,500

c. To find the profit of soccer balls, subtract the expenses from the income. See table below.

Number of Soccer Balls	Profit of Soccer Balls
500	500 − 1,250 = −750
1,000	1,000 − 1,500 = −500
3,000	3,000 − 2,500 = 500

d. The break-even point is at 2,000 soccer balls; the income and expenses are both \$2,000.

e. Income = 1 times the number of soccer balls or

$I = 1n$

Expenses = 1,000 + 0.5 times number of soccer balls or

$E = 1{,}000 + 0.5n$

Profit = Income − Expenses or

$P = 1n - (1{,}000 + 0.5n)$ or

$P = n - 1{,}000 - 0.5n$, or

$P = 0.5n - 1000$

f. $−125 or a loss of $125; (Figure 1).

g. 22,000 : Profit =

$$-1{,}000 + 0.5(\text{number of soccer balls})$$

$$\$10{,}000 = -1{,}000 + 0.5n$$

$$10{,}000 + 1{,}000 = 1{,}000 + 1{,}000 + 0.5n$$

$$11{,}000 = 0.5n$$

$$\frac{11{,}000}{0.5} = \frac{0.5n}{0.5}$$

$$22{,}000 = n$$

The number of soccer balls produced and sold if the profit is $10,000 is 22,000

19. One possible solution:

$$7x + 15 = 12x + 5$$

$$7x - 7x + 15 = 12x - 7x + 5$$

$$15 = 5x + 5$$

$$15 - 5 = 5x + 5 - 5$$

$$10 = 5x$$

$$\frac{10}{5} = \frac{5x}{5}$$

$$2 = x$$

Check:

$$7x + 15 = 12x + 5$$

$$7(2) + 15 = 12(2) + 5$$

$$14 + 15 = 24 + 5$$

$$29 = 29$$

20. $x = 2$; The solution is the same as Exercise 19 because the Commutative Property does not change the value of the variables when solving an equation.

21. One possible solution:

$$-3x + 5 = 2x - 10$$

$$-3x - 2x + 5 = 2x - 2x - 10$$

$$-5x + 5 = -10$$

$$-5x + 5 - 5 = -10 - 5$$

$$-5x = -15$$

$$\frac{-5x}{-5} = \frac{-15}{-5}$$

$$x = 3$$

Check:

$$-3x + 5 = 2x - 10$$

$$-3(3) + 5 = 2(3) - 10$$

$$-9 + 5 = 6 - 10$$

$$-4 = -4$$

22. One possible method:

$$14 - 3x = 1.5x + 5$$

$$14 - 3x - 14 = 1.5x + 5 - 14$$

$$-3x = 1.5x - 9$$

$$-3x - 1.5x = 1.5x - 9 - 1.5x$$

$$(-3 - 1.5)x = -9$$

$$-4.5x = -9$$

$$x = 2$$

Check:

$$14 - 3(2) = 1.5(2) + 5$$

$$8 = 3 + 5$$

$$8 = 8$$

Figure 1

Number of Soccer Balls	Income = # of Soccer Balls I = 1n	Expenses = 1,000 + 0.5 (# of soccer balls)	Profit = Income − Expenses or $P = 0.5n - 1{,}000$
1,750	1,750	1,000 + 0.5(1,750) = 1,000 + 875 = 1,875	1,750 − 1,875 = −125, or 0.5(1,750) − 1,000 = 875 − 1,000 = −125

23. One possible solution:

$$
\begin{aligned}
9 - 4x &= \frac{(3 + x)}{2} \\
2(9 - 4x) &= 2 \times \frac{(3 + x)}{2} \\
18 - 8x &= 3 + x \\
18 - 8x - x &= 3 + x - x \\
18 - 9x &= 3 \\
18 - 9x - 18 &= 3 - 18 \\
-9x &= -15 \\
\frac{-9x}{-9} &= \frac{-15}{-9} \\
x &= 1\tfrac{2}{3}
\end{aligned}
$$

Check:

$$
\begin{aligned}
9 - 4(1\tfrac{2}{3}) &= \frac{3 + (1\tfrac{2}{3})}{2} \\
9 - 6\tfrac{2}{3} &= \frac{4\tfrac{2}{3}}{2} \\
2\tfrac{1}{3} &= 2\tfrac{1}{3}
\end{aligned}
$$

24. One possible solution:

$$
\begin{aligned}
-3(x + 5) &= \frac{(2x - 10)}{3} \\
-3x - 15 &= \frac{(2x - 10)}{3} \\
3(-3x - 15) &= 3 \times \frac{(2x - 10)}{3} \\
-9x - 45 &= 2x - 10 \\
-9x - 2x - 45 &= 2x - 2x - 10 \\
-11x - 45 &= -10 \\
-11x - 45 + 45 &= -10 + 45 \\
-11x &= 35 \\
\frac{-11x}{-11} &= \frac{35}{-11} \\
x &= -3\tfrac{2}{11}
\end{aligned}
$$

Check:

$$
\begin{aligned}
-3(-3\tfrac{2}{11} + 5) &= \frac{2x - 10}{3} \\
-3(1\tfrac{9}{11}) &= \frac{2(-3\tfrac{2}{11}) - 10}{3} \\
\frac{-60}{11} &= \frac{\frac{-70}{11} - 10}{3} \\
\frac{-60}{11} &= \frac{\frac{-180}{11}}{3} \\
\frac{-180}{11} &= \frac{-180}{11}
\end{aligned}
$$

2. Marcel and Kirsten each try to simplify the following equation:
$P = (1{,}000 + 5c + 15a) - (500 + 6c + 8.50a + 250)$

They are both incorrect. Study the steps in their reasoning and identify their mistakes.

a. Marcel

$$\begin{aligned} P &= (1{,}000 + 5c + 15a) - (500 + 6c + 8.50a + 250) \\ &= 1{,}000 + 5c + 15a - 500 + 6c + 8.50a + 250 \\ &= 1{,}000 - 500 + 250 + 5c + 6c + 15a + 8.50a \\ &= 750 + 11c + 23.50a \quad \textit{incorrect answer} \end{aligned}$$

b. Kirsten

$$\begin{aligned} P &= (1{,}000 + 5c + 15a) - (500 + 6c + 8.50a + 250) \\ &= 1{,}000 + 5c + 15a - 500 - 6c - 8.50a - 250 \\ &= 1{,}000 - 500 - 250 + 5c - 6c + 15a - 8.50a \\ &= 250 + c + 6.50a \quad \textit{incorrect answer} \end{aligned}$$

3. According to the equation $V = 200 + 50(T - 70)$, the number of visitors to a park depends on the day's high temperature T (in Fahrenheit). Suppose 1,000 people visited the park one day. Predict that day's high temperature.

Homework Help Online
PHSchool.com
For: Help with Exercise 3
Web Code: ape-6303

For Exercises 4–7, solve each equation for x using the techniques that you developed in Problem 3.1. Check your solutions.

4. $10 + 2(3 + 2x) = 0$

5. $10 - 2(3 + 2x) = 0$

6. $10 + 2(3 - 2x) = 0$

7. $10 - 2(3 - 2x) = 0$

8. The two companies from Problem 3.2 decide to lower their costs for a Fourth of July sale. The equations below show the lower estimated costs C (in dollars) of buying and installing N border tiles.

Cover and Surround It: $C_C = 750 + 22(N - 12)$

Tile and Beyond: $C_T = 650 + 30(N - 10)$

a. Without using a table or graph, find the number of tiles for which the cost estimates from the two companies are equal.

Sky ?? w/ symbols

b. How can you check that your solution is correct?

c. Explain how a graph or table could be used to find the number of tiles for which the costs are equal.

d. For what numbers of tiles is *Tile and Beyond* cheaper than *Cover and Surround It*? Explain your reasoning.

e. Write another expression that is equivalent to the expression for *Tile and Beyond's* cost estimate (C_T). Explain what information the variables and numbers represent.

9. The school choir from Problem 3.1 has the profit plan $P = 5s - (100 + 2s)$. The school band also sells greeting cards. The equation for the band's profit is $P = 4s - 2(10 + s)$. Find the number of boxes that each group must sell to have equal profits.

For Exercises 10–17, solve each equation for *x* without using tables or graphs. Check your solutions.

10. $8x + 16 = 6x$

11. $8(x + 2) = 6x$

12. $6 + 8(x + 2) = 6x$

13. $4 + 5(x + 2) = 7x$

14. $2x - 3(x + 6) = -4(x - 1)$

15. $2 - 3(x + 4) = 9 - (3 + 2x)$

16. $2.75 - 7.75(5 - 2x) = 26$

17. $\frac{1}{2}x + 4 = \frac{2}{3}x$

18. Write each product in expanded form.

a. $(x - 2)(x + 2)$

b. $(x - 5)(x + 5)$

c. $(x - 4)(x + 4)$

d. $(x - 12)(x + 12)$

SWS – Ace pages 46, 47 answers

2. a. When evaluating the second set of parentheses, Marcel distributed the minus sign to the 500, but not to the other three terms.

   b. Kirsten combined $5c - 6c$ and got c instead of $-c$.

3. $1{,}000 = 200 + 50(T - 70)$
   $1{,}000 = 200 + 50T - 3500$
   $1{,}000 = 50T - 3300$
   $4{,}300 = 50T$
   $86 = T$

   Other logical arguments are possible. Students might choose to solve this problem with a table or graph.

4. -4; $10 + 2(3 + 2x) = 0$
   $10 + 6 + 4x = 0$
   $16 + 4x = 0$
   $16 + 4x - 16 = 0 - 16$
   $4x = -16$
   $x = -4$

5. 1; $10 - 2(3 + 2x) = 0$
$$10 - 6 - 4x = 0$$
$$4 - 4x = 0$$
$$4 - 4x - 4 = 0 - 4$$
$$-4x = -4$$
$$x = 1$$

6. 4; $10 + 2(3 - 2x) = 0$
$$10 + 6 - 4x = 0$$
$$16 - 4x = 0$$
$$16 - 4x - 16 = 0 - 16$$
$$-4x = -16$$
$$x = 4$$

7. –1; $10 - 2(3 - 2x) = 0$
$$10 - 6 + 4x = 0$$
$$4 + 4x = 0$$
$$4 + 4x - 4 = 0 - 4$$
$$4x = -4 \text{, so } x = -1$$

8. a. 17;
$$750 + 22(N - 12) = 650 + 30(N - 10)$$
$$750 + 22N - 264 = 650 + 30N - 300$$
$$486 + 22N = 350 + 30N$$
$$486 - 350 + 22N = 350 - 350 + 30N$$
$$136 + 22N = 30N$$
$$136 + 22N - 22N = 30N - 22N$$
$$\frac{136}{8} = \frac{8N}{8}$$
$$17 = N$$

NOTE: for this context to make sense, $N \geq 12$.

b. One possible way to check that the solution is correct is to put the value for N, 17, into each equation, solve for cost and see if both equations have the same value.

c. The point on the graphs at which the two lines intersect is the number of tiles for which the cost estimates are equal. Using a table one would look for the number of tiles for which both companies have the same cost values.

d. *Tile and Beyond* and *Cover and Surround It* have equal cost estimates when the number of tiles is 17.

e. $C = 650 + 30(N - 10)$
$C = 650 + 30N - 300$
$C = 30N + 350$

The 30 means that each tile costs 30 dollars and the 350 is the start-up cost.

9. 80 boxes; Students may graph the two equations and find the x-coordinate of the intersection point. Or they may make a table for each equation and find for which x-coordinate the profits are equal. If students solve symbolically:
$$4s - 2(10 + s) = 5s - (100 + 2s)$$
$$4s - 20 - 2s = 5s - 100 - 2s$$
$$2s - 20 = 3s - 100$$
$$2s - 20 + 100 = 3s - 100 + 100$$
$$2s + 80 = 3s$$
$$2s + 80 - 2s = 3s - 2s$$
$$80 = s$$

10. One possible answer method:
$$8x + 16 = 6x$$
$$8x + 16 - 8x = 6x - 8x$$
$$16 = -2x$$
$$-8 = x$$

Check:
$$8(-8) + 16 = 6(-8)$$
$$-64 + 16 = -48$$
$$-48 = -48$$

11. One possible answer method:
$$8(x + 2) = 6x$$
$$8x + 16 = 6x$$
$$8x + 16 - 8x = 6x - 8x$$
$$16 = -2x$$
$$-8 = x$$

Check:
$$8(-8 + 2) = 6(-8)$$
$$8(-6) = -48$$
$$-48 = -48$$

12. One possible answer method:
$$6 + 8(x + 2) = 6x$$
$$6 + 8x + 16 = 6x$$
$$22 + 8x = 6x$$
$$22 + 8x - 8x = 6x - 8x$$
$$22 = -2x$$
$$-11 = x$$

Check:
$$6 + 8(x + 2) = 6x$$
$$6 + 8(-11 + 2) = 6(-11)$$
$$6 + 8(-9) = -66$$
$$6 + (-72) = -66$$
$$-66 = -66$$

13. One possible answer method:

$$\begin{aligned} 4 + 5(x + 2) &= 7x \\ 4 + 5x + 10 &= 7x \\ 14 + 5x - 5x &= 7x - 5x \\ 14 &= 2x \\ 7 &= x \end{aligned}$$

Check:

$$\begin{aligned} 4 + 5(7 + 2) &= 7(7) \\ 4 + 5(9) &= 49 \\ 49 &= 49 \end{aligned}$$

14. One possible answer:

$$\begin{aligned} 2x - 3(x + 6) &= -4(x - 1) \\ 2x - 3x - 18 &= -4x + 4 \\ -x - 18 &= -4x + 4 \\ -x - 18 - 4 &= -4x + 4 - 4 \\ -x - 22 &= -4x \\ -x - 22 + x &= -4x + x \\ -22 &= -3x \\ \tfrac{22}{3} &= x \text{ or } x = 7\tfrac{1}{3} \end{aligned}$$

Check:

$$\begin{aligned} 2(\tfrac{22}{3}) - 3(\tfrac{22}{3} + 6) &= -4(\tfrac{22}{3} - 1) \\ \tfrac{44}{3} - 22 - 18 &= -\tfrac{88}{3} + 4 \\ \tfrac{44}{3} - 40 &= -\tfrac{88}{3} + \tfrac{12}{3} \\ \tfrac{44}{3} - \tfrac{120}{3} &= -\tfrac{88}{3} + \tfrac{12}{3} \\ -\tfrac{76}{3} &= -\tfrac{76}{3} \end{aligned}$$

15. One possible answer:

$$\begin{aligned} 2 - 3(x + 4) &= 9 - (3 + 2x) \\ 2 - 3x - 12 &= 9 - 3 - 2x \\ -3x - 10 &= 6 - 2x \\ -3x - 10 - 6 &= 6 - 2x - 6 \\ -3x - 16 &= -2x \\ -3x - 16 + 3x &= -2x + 3x \\ -16 &= x \end{aligned}$$

Check:

$$\begin{aligned} 2 - 3(-16 + 4) &= 9 - (3 + 2(-16)) \\ 2 - 3(-12) &= 9 - (3 - 32) \\ 2 + 36 &= 9 - (-29) \\ 38 &= 38 \end{aligned}$$

16. One possible answer:

$$\begin{aligned} 2.75 - 7.75(5 - 2x) &= 26 \\ 2.75 - 38.75 + 15.5x &= 26 \\ -36 + 15.5x &= 26 \\ -36 + 36 + 15.50x &= 26 + 36 \\ 15.50x &= 62 \\ x &= 4 \end{aligned}$$

Check:

$$\begin{aligned} 2.75 - 7.75(5 - 2(4)) &= 26 \\ 2.75 - 7.75(5 - 8) &= 26 \\ 2.75 - 7.75(-3) &= 26 \\ 2.75 + 23.25 &= 26 \end{aligned}$$

17.

$$\begin{aligned} \tfrac{1}{2}x + 4 &= \tfrac{2}{3}x \\ \tfrac{1}{2}x + 4 - \tfrac{1}{2}x &= \tfrac{2}{3}x - \tfrac{1}{2}x \\ 4 &= \tfrac{1}{6}x \\ 4 \div \tfrac{1}{6} &= x \\ 24 &= x \end{aligned}$$

Check:

$$\begin{aligned} \tfrac{1}{2}(24) + 4 &= \tfrac{2}{3}(24) \\ 12 + 4 &= 16 \\ 16 &= 16 \end{aligned}$$

18. **a.** $x^2 - 4$ **b.** $x^2 - 25$
c. $x^2 - 16$ **d.** $x^2 - 144$

19. **a.** $(x + 1)(x + 4)$ **b.** $(x + 2)(x + 4)$
c. $(x - 5)(x - 2)$ **d.** $x(x + 7)$
e. $(x - 1)(x + 6)$ **f.** $(2x + 3)(x - 4)$
g. $(x + 1)(x - 8)$ **h.** $x(x - 5)$

20. **a.** $(x + 4)(x - 4)$ **b.** $(x + 6)(x - 6)$
c. $(x - 7)(x + 7)$ **d.** $(x + 20)(x - 20)$
e. $(x - 8)(x + 8)$ **f.** $(x - 12)(x + 12)$

21.

$$\begin{aligned} x^2 + 1.5x &= 0 \\ x(x + 1.5) &= 0 \\ x = 0 &\text{ or } x = -1.5 \end{aligned}$$

22.

$$\begin{aligned} x^2 + 6x + 8 &= 0 \\ (x + 2)(x + 4) &= 0 \\ x = -2 &\text{ or } x = -4 \end{aligned}$$

23.

$$\begin{aligned} 8x - x^2 &= 0 \\ x(8 - x) &= 0 \\ x = 0 &\text{ or } x = 8 \end{aligned}$$

Shapes of Algebra

*

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	**6-8 Performance Expectations**
2.1 Graphs of Linear Systems	1			
2.2 Linear Inequalities	1			**8.1.B Solve one- and two-step linear inequalities and graph the solutions on the number line.**
2.3 Solving Linear Inequalities	1			
ACE problems Investigation 2 select from #1-42 not # 35	1			
Check Up #1	1			**8.1.D Determine the slope and y-intercept of a linear function described by a symbolic expression, table, or graph.**
On-line Lessons Writing and Solving Inequalities (AL-c) ***13.1 Writing Inequalities*** ***13.2 Properties of Inequalities*** ***13.3 Solving Inequalities***	2			
3.1 Solving Equations with 2 variables	1			**8.1.F Solve single- and multi-step word problems involving linear functions and verify the solutions.**
3.2 Connecting y = mx + b and ax + by = c	1			
3.3 Intersection of Lines	2			
(CMP2) ACE problems Investigation 3 select from #1 -41, 48,49	1			Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
Total Instructional Days	**14**			

Contents in Shapes of Algebra

Topic 13: Writing and Solving Inequalities

for use after *Moving Straight Ahead* **Investigation 3**

You just explored what it means for two quantities to be equal. When you write an equation you are comparing the value of two equal quantities. Sometimes you need to compare two quantities that are not equal.

An **inequality** is a mathematical sentence that compares the values of two expressions that are not equal. Instead of using an equal sign, you use an inequality symbol.

Inequality Symbols

$<$	less than
$>$	greater than
$\leq$	less than or equal to
$\geq$	greater than or equal to
$\neq$	not equal to

Problem 13.1 Writing Inequalities

A. For each situation, first define a variable. Then represent the situation with an inequality.

1. The height of a child must be at least 48 inches to ride a roller coaster.
2. The speed limit on the road is less than or equal to 45 miles per hour.
3. A piece of luggage must be less than 60 pounds.
4. You must be at least 13 years old to see a movie rated PG-13.

Exercises

For each situation, define a variable. Then write an inequality to model each situation.

1. Ana has a car. She wants to limit herself to driving at most 500 miles per month.
2. The Simon family's car emits 0.75 pounds of CO_2 per mile. It emits 2 pounds of CO_2 when it is started. The Simons want to limit their emissions to at most 100 pounds of CO_2 per use of the car.
3. An online discount costs \$50 per month. It decreases \$2 for every person you recommend to sign-up. You want to keep the total cost below \$35.

You know that to maintain an equality, you can add, subtract, multiply or divide both sides of the equality by the same number.

Problem 3.2 Properties of Inequalities

A. Consider the inequality $6 < 20$.

1. Does the inequality remain true when you add or subtract each side by the same number?
2. Does it remain true when you multiply or divide by a positive number? A negative number?
3. What seems to be true about properties of inequalities?

B. Sally did the following work to solve the inequality $-4x + 2 < 10$.

$-4x + 2 < 10$	
$-4x + 2 - 2 < 10 - 2$	Subtract 2 from each side.
$-4x < 8$	Simplify each side.
$\frac{-4x}{-4} > \frac{8}{-4}$	Divide each side by -4. Reverse the inequality symbol.
$x > -2$	Simplify.

Is she correct? Explain.

Problem 3.3 Solving Inequalities

Each month the expenses of Fabulous Fabian's Bakery from Problem 3.5 in *Moving Straight Ahead* must be less than a set amount. Recall that the expenses E to make n cakes in a month is represented by $E = 825 + 3.25n$.

A. Fabian needs to keep expenses this month less than \$2,200.

1. Write an inequality using this information.
2. Solve your inequality. What does this mean for Fabian's bakery?

B. Check to make sure your answer is reasonable.

Exercises

Solve each inequality.

1. $3x + 17 < 47$
2. $14x - 23 < 5x + 13$
3. $43 < 8t - 9$
4. $182 < -4m + 2$
5. $-6c + 9 < 25$
6. $3{,}985 + 59 < 995 + 14.95d$
7. Vince finds out that his family's SUV emits an average of 1.25 pounds of CO_2 per mile. Suppose Vince's family wants to limit CO_2 to at most 600 pounds per month.
   - **a.** Write an inequality using this information.
   - **b.** Solve your inequality. What does the solution mean for the Vince's family?
   - **c.** Check to make sure your answer is reasonable.

Topic 13: Writing and Solving Inequalities

At a Glance

PACING 1 day

Mathematical Goals

- Write inequalities to represent situations
- Solve inequalities in one variable

Teaching Guide

Vocabulary

- inequality

Students should have a clear understanding of equations before they start working with inequalities, as many of the same rules apply. While students should be familiar with the similarities between equations and inequalities, it is also crucial that they recognize the differences. Remind students throughout the lesson to use the correct inequality symbols since some students may use equal signs out of habit.

Many students will also need to practice writing verbal statements as algebraic expressions. Sufficient time should be spent familiarizing students with the meanings of the inequality symbols. One possible classroom activity is to divide the students into pairs and instruct them to take turns writing situations for their partner to translate into algebraic expressions.

When solving inequalities, many students struggle with solving by multiplication and division. You may choose to do several examples as a class to highlight the importance of reversing the inequality sign when multiplying or dividing by a negative number.

After Problem 13.1, ask:

- *What are some common phrases that are used to describe "greater than or equal to"? To describe "less than or equal to"?*
- *How would you rewrite an inequality with the variable on the opposite side of the inequality symbol?*

Explain to students what it means for a number to satisfy an inequality, or be a solution to the inequality. Have students find several different solutions to the same inequality. Point out to students the difference between the solutions for pairs of inequalities like $x < 2$ and $x \leq 2$. Then have students discuss what the solutions of the inequality mean in a real-world situation.

Summarize the example before Problem 13.2 by asking:

- *What values could you use to check your solution to the inequality?*

During Problem 13.2, ask:

- *What information does the number 825 represent?*
- *What information does the number 3.25 represent?*
- *For which variable in the inequality should you substitute 2,200?*

Homework Check

When reviewing Exercise 6, ask:

- *What was your first step in solving this inequality?*
- *Could you still get the correct answer without doing this first?*

Assignment Guide for Topic 13

Core Problem 13.1 Exercises 1–3, Problem 13.2–13.3 Exercises 1–6
Advanced Problem 13.2–13.3 Exercise 7

Answers to Topic 13

Problem 13.1

A. **1.** Let h = the child's height in inches; $h \geq 48$.

2. Let s = speed in miles per hour; $s \leq 45$.

3. Let w = the weight of the piece of luggage in pounds; $w < 60$.

4. Let a = your age in years; $a \geq 13$.

Exercises

1. Let m = the number of miles Ana drives; $m \leq 500$
2. Let m = the number of miles the Simons drive; $2 + 0.75m \leq 100$
3. Let p = the number of people you recommend; $50 - 2p < 35$

Problem 13.2

A. **1.** yes

2. yes; no

3. Inequalities remain true when you add or subtract each side by the same number, and when you multiply or divide by a positive number.

B. Yes; Sally reversed the inequality symbol when she divided each side by a negative number.

Problem 13.3

A. **1.** $2{,}200 > 825 + 3.25n$

2. $n < 423.08$; the bakery must make fewer than 423 cakes this month.

B. Check students' work.

Exercises

1. $x < 10$
2. $x < 4$
3. $t > 6.5$
4. $m < -45$
5. $c > -2\frac{2}{3}$
6. $d > 203.95$
7. **a.** $1.25m \leq 600$

   b. $m \leq 480$; Vince's family must drive no more than 480 miles per month.

   c. Check students' work; students should show that they checked at least one value that makes the inequality true, as well as one that makes the inequality false.

Looking for Pythagoras

Investigation / Lesson / Assessments	# Of Days	Resource Location	Follow Up?	**6-8 Performance Expectations**
1.1 Driving Around Euclid, p.5	0.5		Should do	**8.2.E** *Quickly* **recall the square roots of the perfect squares from 1 through 225 and estimate the square roots of other positive numbers.**
1.2 Planning Emergency Routes, p.9	0.5		Should do	
1.3 Planning Parks, p.10	1		Optional	
Inv. 1 Reflections, p.16	1			
2.2 Looking for Squares, p.18	3		Must do	**8.2.F** *Demonstrate the Pythagorean Theorem and its converse and* **apply them to solve problems.**
2.3 Finding Lengths, p. 20 & Square Root Operations in Add'l. Practice Inv. 2 #5-11	2		Must do	
Estimating Square Roots **Worksheet**	1	binder		**8.2.G Apply the Pythagorean Theorem to determine the distance between two points on the coordinate plane.**
Perfect Squares **Worksheet**	1	binder		
Inv. 2 Reflections, p.26	1			
3.1 Discovering the Pythagorean Theorem, p.27	2		Must do	*8.4.D Identify rational and irrational numbers.*
3.2 Puzzling Through a Proof, p.29	2		Must do	
3.3 Finding Distance, p. 31	1		Should do	Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
3.4 Measuring the Egyptian Way, p.32	2		Must do #1	
ACE #8-11 p. 35 and extend to finding distance between two coordinate pairs	1			
Inv. 3 Reflections, p.40	1			
Check-Up	1			*** When completing ACE questions associated with problems 5.1 – 5.3, be sure to identify whether the numbers given are rational or irrational and explain how they know. Also be sure to have kids practice placing rational and irrational numbers on a number line.**
4.1 Stopping Sneaky Sally, p. 41 & Inv. 4 ACE #1, 3, 5, 6 pages 46—49	1		Should do	
5.1 Analyzing the Wheel of Theodorus pg. 53 *	1			
5.2 Representing Fractions as Decimals pg. 56 *	1			
5.3 Exploring Repeating Decimals pg. 57 *	1			
Rational vs. Irrational Numbers **Worksheet**	1	binder		
Inv. 5 Reflections pg. 63	1			
Review for Unit Assessment	1			
Unit Assessment	1			
Total Instructional Days:	**29**			

Contents in Looking for Pythagoras

- Estimating Square Roots Worksheet : 2 pages
- Estimating Square Roots Answer Key
- Perfect Squares Worksheet : 1 page
- Perfect Squares Answer Key
- Rational vs. Irrational Numbers Worksheet : 2 pages
- Irrational Numbers Answer Key

Name: ______________________________ Date: __________ Period: _____

Estimating Square Roots

Definitions:

Perfect Square: A number whose square root is an integer.
Square root of a number: One of two identical factors of a number.
Radical sign: $\sqrt{}$

Perfect Squares

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2
1	4	9	16	25	36	49	64	81	100	121	144

1. Estimate $\sqrt{10}$ to the nearest tenth. ____________

   9 is the closest perfect square less than 10 and 16 is the closest perfect square greater than 10.

   $\sqrt{9}$ = ________ and $\sqrt{16}$ = __________ So, $\sqrt{10}$ is between the whole

   numbers _____ and _____.

2. When estimating square roots, you want to be a little more precise than saying the $\sqrt{10}$ is between 3 and 4.

   Is it closer to 3 or closer to 4? ________

   Now estimate $\sqrt{10}$ to the nearest tenth. ________

3. Estimate $\sqrt{39}$ to the nearest tenth.

   ______ is the closest perfect square less than 39 and ______ is the closest perfect square greater than 39.

   $\sqrt{\_\_\_\_\_}$ = ________ and $\sqrt{\_\_\_\_\_}$ = ________

   So, $\sqrt{39}$ is between ________ and ______.

   $\sqrt{39}$ estimated to the nearest tenth is ______

Problem	Between	Answer rounded to the nearest tenth
$\sqrt{20}$	Square roots: $\sqrt{16}$ and $\sqrt{25}$ Whole numbers: ________ and ________	
$\sqrt{52}$	Square roots: $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$ Whole numbers: ________ and ________	
$\sqrt{8}$	Square roots: $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$ Whole numbers: ________ and ________	
$\sqrt{40}$	Square roots: $\sqrt{\_\_\_\_}$ and $\sqrt{\_\_\_\_}$ Whole numbers: ________ and ________	

A. 1. Find the side lengths of square with areas of 1,9,16, 25, and 36.

Area of the Square	Side Length
1	
9	
16	
25	
36	

2. Find the values of:

$\sqrt{4}$ = ________ $\sqrt{16}$ = ________ $\sqrt{36}$ = ________ $\sqrt{81}$ = ________

B. a. What is the area of a square with a side length of 12 units? Draw a model.

b. What is the area of a square with a side length of 25 units. Draw a model.

Name: Key Date: ____________ Period: ______

Estimating Square Roots

Definitions:

Perfect Square: A number whose square root is an integer.
Square root of a number: One of two identical factors of a number.
Radical sign: $\sqrt{\ }$

Perfect Squares

1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2	11^2	12^2
1	4	9	16	25	36	49	64	81	100	121	144

1. Estimate $\sqrt{10}$ to the nearest tenth. $\approx$ 3.1

   9 is the closest perfect square less than 10 and 16 is the closest perfect square greater than 10.

   $\sqrt{9}$ = 3 and $\sqrt{16}$ = 4 So, $\sqrt{10}$ is between the whole numbers 3 and 4.

2. When estimating square roots, you want to be a little more precise than saying the $\sqrt{10}$ is between 3 and 4.

   Is it closer to 3 or closer to 4? 3

   Now estimate $\sqrt{10}$ to the nearest tenth. 3.1

3. Estimate $\sqrt{39}$ to the nearest tenth.

   36 is the closest perfect square less than 39 and 49 is the closest perfect square greater than 39.

   $\sqrt{36}$ = 6 and $\sqrt{49}$ = 7

   So, $\sqrt{39}$ is between 6 and 7.

   $\sqrt{39}$ estimated to the nearest tenth is 6.2 or 6.3

$\sqrt{36}$ $\sqrt{37}$ $\sqrt{38}$ $\boxed{\sqrt{39}}$ $\sqrt{40}$ $\sqrt{41}$ $\sqrt{42}$ $\sqrt{43}$ $\sqrt{44}$ $\sqrt{45}$ $\sqrt{46}$ $\sqrt{47}$ $\sqrt{48}$ $\sqrt{49}$

6.5

Problem	Between	Answer rounded to the nearest tenth
$\sqrt{20}$	Square roots: $\sqrt{16}$ and $\sqrt{25}$ Whole numbers: 4 and 5	4.4
$\sqrt{52}$	Square roots: $\sqrt{49}$ and $\sqrt{64}$ Whole numbers: 7 and 8	7.3 or 7.2
$\sqrt{8}$	Square roots: $\sqrt{4}$ and $\sqrt{9}$ Whole numbers: 2 and 3	2.8 or 2.9
$\sqrt{40}$	Square roots: $\sqrt{36}$ and $\sqrt{49}$ Whole numbers: 6 and 7	6.3 or 6.4

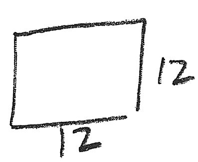
A. 1. Find the side lengths of square with areas of 1,9,16, 25, and 36.

Area of the Square	Side Length
1	$\sqrt{1}=1$
9	$\sqrt{9}=3$
16	$\sqrt{16}=4$
25	$\sqrt{25}=5$
36	$\sqrt{36}=6$

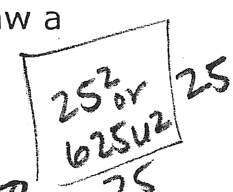
2. Find the values of:

$\sqrt{4}$ = 2 $\sqrt{16}$ = 4 $\sqrt{36}$ = 6 $\sqrt{81}$ = 9

B. a. What is the area of a square with a side length of 12 units? Draw a model.



$12 \cdot 12 = 144u^2$



b. What is the area of a square with a side length of 25 units. Draw a model.

NAME ______________________________ **DATE** ________ **PERIOD** _____

Perfect Squares

List the first twenty perfect squares without using a calculator:

$1^2 =$

$2^2 =$

$3^2 =$

$4^2 =$

$5^2 =$

$6^2 =$

$7^2 =$

$8^2 =$

$9^2 =$

$10^2 =$

$11^2 =$

$12^2 =$

$13^2 =$

$14^2 =$

$15^2 =$

$16^2 =$

$17^2 =$

$18^2 =$

$19^2 =$

$20^2 =$

Use the perfect squares above to estimate the following square roots:

$\sqrt{5} \approx$

$\sqrt{119} \approx$

$\sqrt{168} \approx$

$\sqrt{353} \approx$

$\sqrt{10} \approx$

$\sqrt{401} \approx$

$\sqrt{201} \approx$

$\sqrt{141} \approx$

$\sqrt{83} \approx$

$\sqrt{277} \approx$

NAME Key DATE ________ PERIOD _____

Perfect Squares

List the first twenty perfect squares without using a calculator:

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$

Use the perfect squares above to estimate the following square roots:

approximates! answers may vary slightly

$\sqrt{5} \approx$ $\sqrt{4}$ & $\sqrt{9}$; 2 & 3; 2.1

$\sqrt{168} \approx$ $\sqrt{144}$ & $\sqrt{169}$; 12 & 13; 12.9

$\sqrt{10} \approx$ $\sqrt{9}$ & $\sqrt{16}$; 3 & 4; 3.1

$\sqrt{201} \approx$ $\sqrt{196}$ & $\sqrt{225}$; 14 & 14; 14.2

$\sqrt{83} \approx$ $\sqrt{81}$ & $\sqrt{100}$; 9 & 10; 9.1

$\sqrt{119} \approx$ $\sqrt{100}$ & $\sqrt{121}$; 10 & 11; 10.9

$\sqrt{353} \approx$ $\sqrt{324}$ & $\sqrt{361}$; 18 & 19; 18.7

$\sqrt{401} \approx$ close to $\sqrt{400}$; 20.1

$\sqrt{141} \approx$ $\sqrt{121}$ & $\sqrt{144}$; 11 & 12; 11.8

$\sqrt{277} \approx$ $\sqrt{256}$ & $\sqrt{289}$; 16 & 17; 16.7

Name ______________________________ Date: ____________________ Period: ______

Looking for Pythagoreans; Rational vs. Irrational Numbers

Some decimals such as 0.5 and 0.375 *terminate* which means they have a limited number of digits.

Other decimals such as 0.3333.... and 0.181818... have a repeating pattern of digits that never ends.

Terminating or repeating decimals are called **rational numbers** because they can be expressed as ratios.

Here are some example of rational numbers written in decimal and ratio (fraction) form:

$0.5 = \frac{1}{2}$ $\quad$ $0.3125 = \frac{5}{16}$ $\quad$ $0.333 = \frac{1}{3}$ $\quad$ $0.181818 = \frac{2}{11}$

Some decimals neither terminate nor do they repeat. Numbers written as decimals that don't terminate and don't repeat are called **irrational numbers**. Irrational numbers can not be written in ratio (fraction) form. Basically, if a number is not a rational number then it is an irrational number.

Here are some examples of irrational numbers:

$\sqrt{2}$ $\quad$ $\sqrt{3}$ $\quad$ π $\quad$ 2.236067978.....

Now look at the list of numbers below. Decide which numbers are rational and which are irrational.

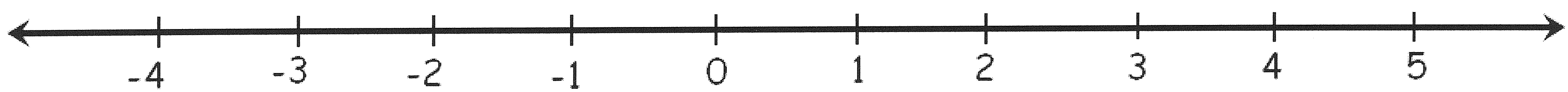
-2.9	$-\frac{16}{9}$	pi	0.343343334...	$\sqrt{5}$	$2.\overline{27}$	$\sqrt{20}$
$-2.\overline{55}$	4	0.001001001	$-\sqrt{2}$	$\sqrt{25}$	0.060606...	4.545

Place all the rational numbers here.	Place all the irrational numbers here.

Rational vs. Irrational Numbers

Now let's place these numbers you identified as rational and irrational on a number line.

-2.9	$-\frac{16}{9}$	pi	0.343343334…	$\sqrt{5}$	$2.\overline{27}$	$\sqrt{20}$
$-2.\overline{55}$	4	0.001001001	$-\sqrt{2}$	$\sqrt{25}$	0.060606…	4.545



Name ___Key___ Date: __________ Period: ______

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Now look at the list of numbers below. Decide which numbers are rational and which are irrational.

$\cancel{-2.9}$ $\quad \cancel{-\frac{16}{9}}$ $\quad$ ~~pi~~ $\quad \cancel{0.343343334...}$ $\quad \cancel{\sqrt{5}}$ $\quad \cancel{2.\overline{27}}$ $\quad \cancel{\sqrt{20}}$

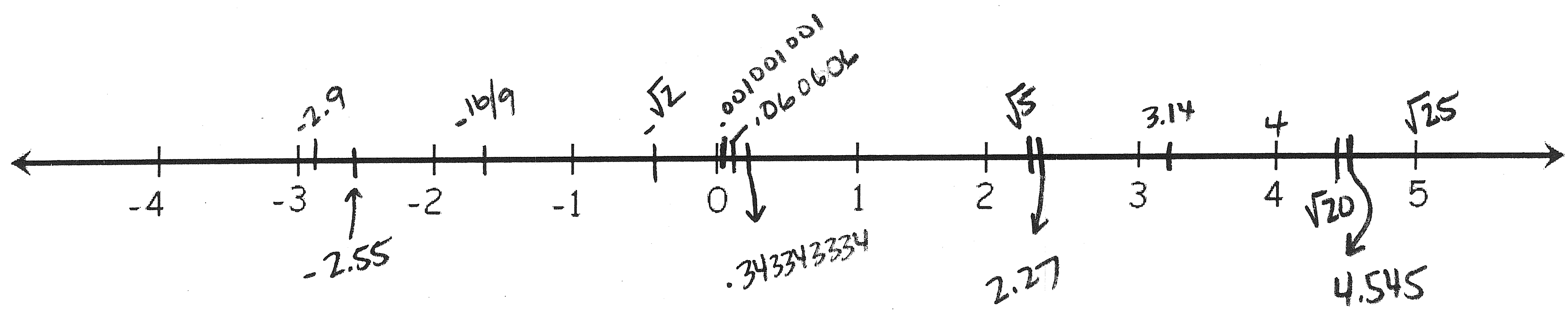
$\cancel{-2.\overline{55}}$ $\quad \cancel{4}$ $\quad \cancel{0.001001001}$ $\quad \cancel{-\sqrt{2}}$ $\quad \cancel{\sqrt{25}}$ $\quad \cancel{0.060606...}$ $\quad \cancel{4.545}$

Place all the rational numbers here.	Place all the irrational numbers here.
$\sqrt{25}$ $\quad 4$ $\quad 2.\overline{27}$ $\quad -2.\overline{55}$ $\quad -2.9$ $\quad 4.545$ $\quad -\frac{16}{9}$ $\quad .001001001$ $\quad .060606$	pi $\quad \sqrt{5}$ $\quad \sqrt{20}$ $\quad -\sqrt{2}$ $\quad .34343334$

Rational vs. Irrational Numbers

Now let's place these numbers you identified as rational and irrational on a number line.

$\cancel{-2.9}$	$\cancel{-\frac{16}{9}}$	$\cancel{\text{pi}}$	$\cancel{0.343343334...}$	$\cancel{\sqrt{5}}$	$\cancel{2.\overline{27}}$	$\cancel{\sqrt{20}}$
$\cancel{-2.\overline{55}}$	$\cancel{4}$	$\cancel{0.001001001}$	$\cancel{-\sqrt{2}}$	$\cancel{\sqrt{25}}$	0.060606...	$\cancel{4.545}$



Kaleidoscopes, Hubcaps & Mirrors

Investigation / Lesson / Assessments	# of Days	Resource Location	Follow Up?	6-8 Performance Expectations
1.1 Reflectional Symmetry, p. 6	1		Optional	**8.2.A Identify pairs of angles as complementary, supplementary, adjacent, or vertical, and use these relationships to determine missing angle measures.** **8.2.B Determine missing angle measures using the relationships among the angles formed by parallel lines and transversals.** **8.2.C** *Demonstrate that the sum of the angle measures in a triangle is 180 degrees, and* **apply this fact to determine the sum of the angle measures of polygons and to determine unknown angle measures.** **8.2.D Represent and explain the effect of one or more translations, rotations, reflections, or dilations (centered at the origin) of a geometric figure on the coordinate plane.** Performance Expectations that will be assessed at the state level appear in **bold text**. *Italicized text* should be taught and assessed at the classroom level.
1.2 Rotational Symmetry, p. 8			Optional	
1.4 Translational Symmetry, p. 12	1		Optional	
Inv. 1 Reflections, p. 23	1			
On-line Lesson Translations on a Grid GM-k	1	On-line lesson		
On-line Lesson Reflections on a Grid GM-l	1	On-line lesson		
Additional Practice w/ Translations and Reflections **(CMP2)** KHM Additional Practice Grade 8 Inv. 2 & 5, Navigations through Data Analysis: Reflection of Images	2	Binder or CMP2 disk		
On-line Lesson Rotations on a Grid GM-m	1	On-line lesson		
Practice with Reflections and Rotations **Worksheet**	1	Binder or CMP2 Disk		
KHM Additional Lesson: Dilations	2	binder		
Practice with Dilations **Worksheet**	1	binder		
Angle Vocab.w/ Parallel Lines **Worksheet**	2	binder		
(CMP2) Grade 6 Investigation 2 Shapes & Designs Skill: Angles and Parallel Lines	1	binder or CMP2 Disk (6)		
On-line Lesson Perpendiculars and Parallels GM-i **1-13 only!**	1	On-line lesson		
(CMP2) Shapes & Designs 6th grade (3.1 Angle Sums of Regular Polygons)	1	CMP2 Disk (6) or binder		
(CMP2) Shapes & Designs 6th grade (3.2 Angle Sums of Any Polygon)	1	CMP2 Disk (6) or binder		
Extra Practice: **(CMP2)** Shapes and Designs Grade 6 Investigation 3 Additional Practice and Skill: Angle Sums and Exterior Angles of Polygons	1	binder or CMP2 Disk (6)		
Review for Unit Assessment	2			
Unit Assessment	1			
Total Instructional Days	**22**			

Contents in Kaleidoscopes, Hubcaps & Mirrors

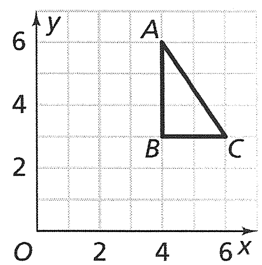
Topic 4: Translations in the Coordinate Plane

for use after ***Shapes and Designs*** **Investigation 4**

A **transformation** is the change in the size, shape, or position of a figure. A **translation** is a transformation in which each point of a figure moves the same distance and in the same direction. This design contains many such figures.

Problem 4.1

A. Copy ΔABC and translate it to $\Delta A'B'C'$ using the steps below.



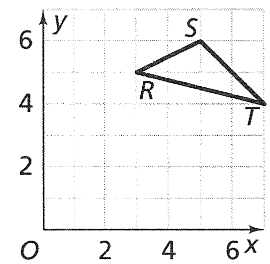
1. From A, count down 2 units and to the left 3 units. Label the new point A'(ay-prime).
2. Find and label points B' and C' by counting down 2 units and left 3 units.
3. Draw $\Delta A'B'C'$.

B. Draw a line from A to A', from B to B', and from C to C'.

1. Compare the length of the three lines.
2. Compare the direction of the three lines.
3. Explain why $\Delta A'B'C'$ is a translation of ΔABC.

Exercises

1. For each of the directions below, copy the graph and translate ΔRST. Label the image $\Delta R'S'T'$.



a. Translate ΔRST up 2 units.

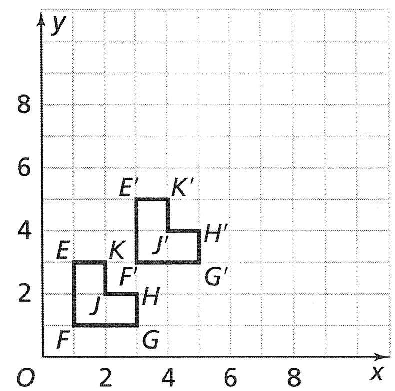
b. Translate ΔRST to the right 2 units.

c. Translate ΔRST to the left 2 units and down 4 units.

d. Translate ΔRST to the right 1 unit and down 1 unit.

e. Translate ΔRST to the left 2 units and up 1 unit.

2. Danielle drew the figures at the right to represent a translation.



a. Describe the translation of point E to point E'.

b. Name the coordinates of an unlabeled point on the bottom figure, then give the coordinates of the translated image of that point.

c. Jeremy says that Danielle only plotted 6 points to do the translation, so that means only 6 points on the original figure were translated. Do you agree with Jeremy?

3. Chee wrote this rule to describe the translation of ΔABC to $\Delta A'B'C'$: $(x, y) \longrightarrow (x + 1, y - 4)$

a. How does Chee's rule use coordinates to translate a figure?

b. Chee drew ΔKLM with vertices at $K(2, 7)$, $L(4, 6)$, and $M(3, 4)$. He then followed his own rule to draw $\Delta K'L'M'$. Draw both of these triangles.

Topic 4: Translations in the Coordinate Plane

At a Glance

PACING 1 day

Mathematical Goals

- Identify the translations used to move a polygon from one location to another in the coordinate plane.
- Explain how translations affect the location of a polygon in the coordinate plane.

Guided Instruction

Among the three main types of transformations your students will study—translations, rotations, and reflections—translations should be the easiest for the students to grasp. Students will find translations relatively easy to model by tracing the outline of a flat shape on graph paper before and after sliding the shape from one location to another. Have students work in pairs for this type of modeling. In those activities where students copy a shape and then draw its prescribed translation, tracing paper can be used to confirm that the original shape and the translated image are congruent.

In the translation problems and exercises, the points being used for the translation are all vertices. You should briefly review the definition of a vertex as the point in a polygon where two sides meet.

Although the translations in this lesson are shown as occurring only in the first quadrant, more advanced students should be encouraged to translate shapes between any two locations in the coordinate plane.

Before Problem 4.1:

- *Describe some of the figures in the design at the top of the page that are repeated as you move from left to right.* (various descriptions of triangles and a square.)

During Problem 4.1, A:

- *Why do you think it is a good idea to name the translated triangle with A', B', and C' instead of just using other letters altogether?* (It makes it easy to match up the original points with the translated points.)

After Problem 4.1:

- *How can you prove that the direction from A to A' is the same as the direction from B to B' and C to C'?* (They all form a diagonal of a 2 unit $\times$ 3 unit rectangle.)
- *How could you make sure that $\Delta A'B'C'$ is the same size and shape as ΔABC?* (Trace one triangle onto tracing paper and see if the shapes match; or measure all the sides and the angles.)
- *What changes when you translate a figure?* (Its position.)

You will find additional work on transformations in the grade 8 unit *Kaleidoscopes, Hubcaps, and Mirrors.*

Vocabulary

- transformation
- translation

Materials

- Labsheets 4. 1, 4ACE Exercise 1

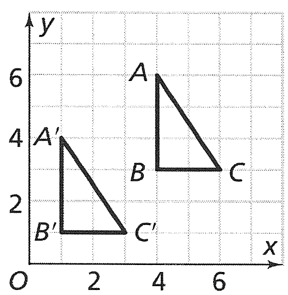
ACE Assignment Guide for Topic 4

Core 1–3

Answers to Topic 4

Problem 4.1

A.



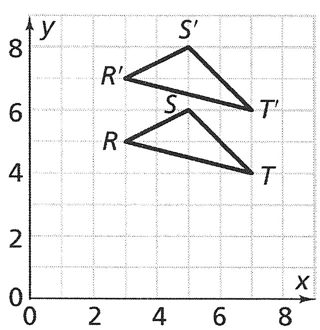
B. **1.** The lines are the same length.

2. The lines are in the same direction.

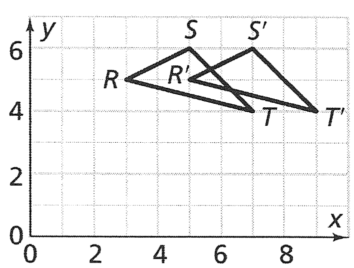
3. Every point on ΔABC moved the same distance and in the same direction.

Exercises

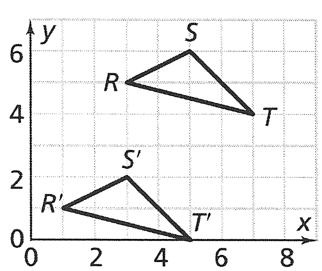
1. a.



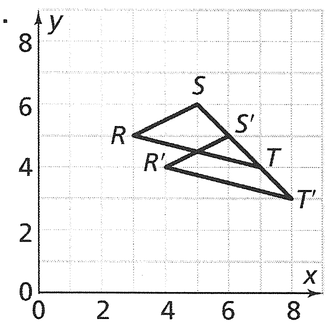
b.



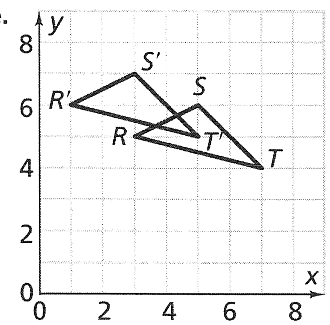
c.



d.



e.



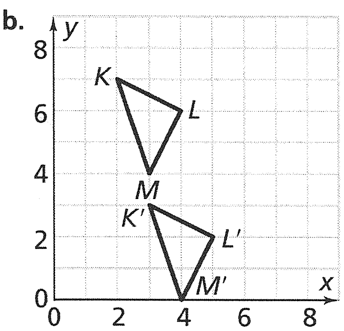
2. a. To the right 2 units then up 2 units.

b. (2, 1); (4, 3)

c. Answers may vary. Sample: Every single point on the original figure was translated. The number of points translated is infinite.

3. a. It says that every point on ΔABC moves 1 unit to the right, then 4 units down.

b.



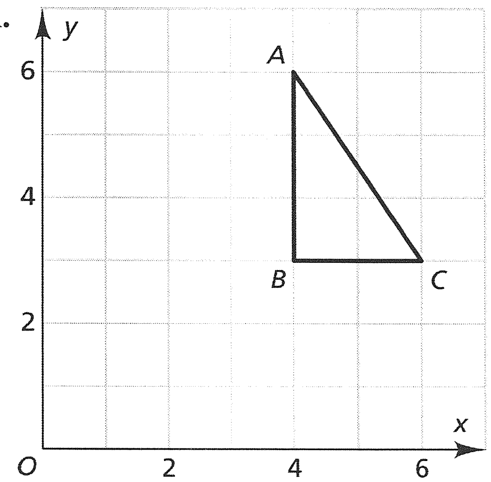
Name ______________________ Date __________ Class __________

Labsheet 4.1

Topic 4

Translations in the Coordinate Plane

A.

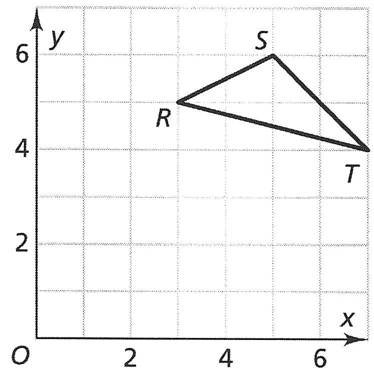


Name ______________________ Date __________ Class __________

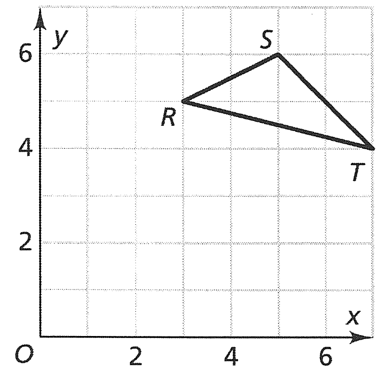
Labsheet 4ACE Exercise 1

Topic 4

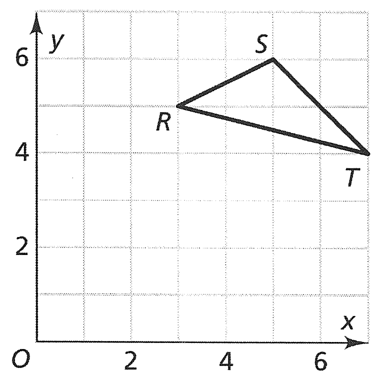
a.



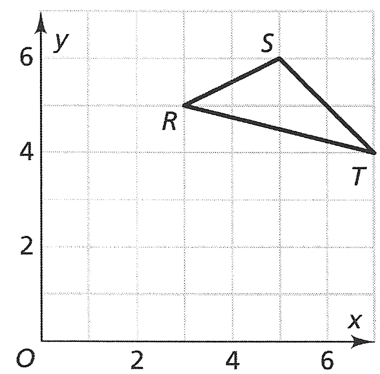
b.



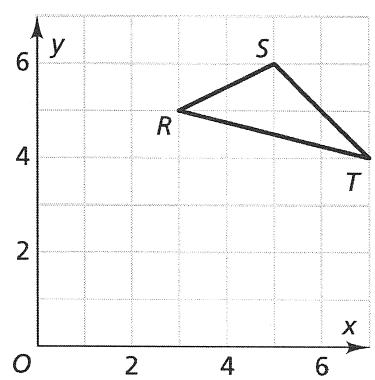
c.



d.



e.



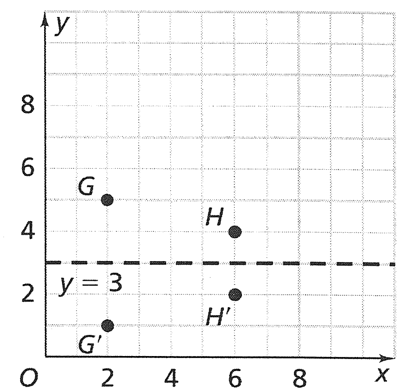
Topic 5: Reflections in the Coordinate Plane

for use after ***Shapes and Designs*** **Investigation 4**

A **reflection** is a transformation that flips an image over a line called the **line of reflection.** If you hold your open hand against the edge of a mirror so that your thumb is facing in your direction, every detail of your real hand appears as a reflected image in the mirror. The edge of the mirror is the line of reflection.

Problem 5.1

A. The reflection of two points across the line $y = 3$ is shown. Point G' (gee-prime) is the reflection of point G. Point H' is the reflection of point H.



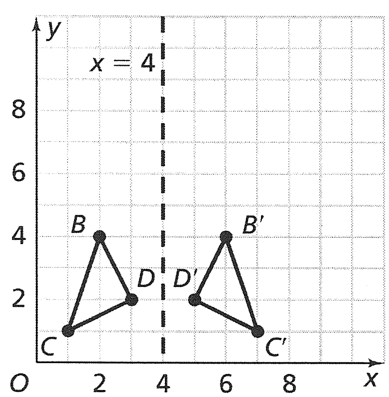
1. What is the shortest distance from G to the line of reflection?

2. Compare your answer to the distance from G' to the line of reflection.

3. Does the same comparison hold true for H and H'?

4. Write a rule for reflecting a point across a line.

B. The reflection of a triangle across the line $x = 4$ is shown below.



1. Fold the graph all the way over along the line $x = 4$. What are you looking at?

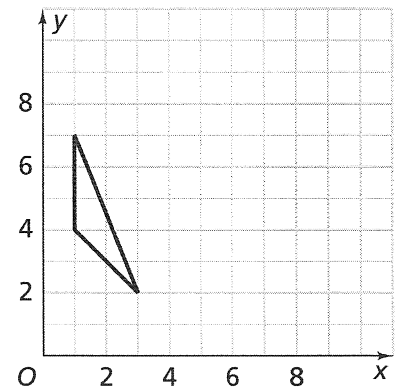
2. What do you notice when you compare the distance from vertex B to the line for $x = 3$ with the distance from vertex B' to the same line?

3. Make the same type of comparison for the remaining vertices.

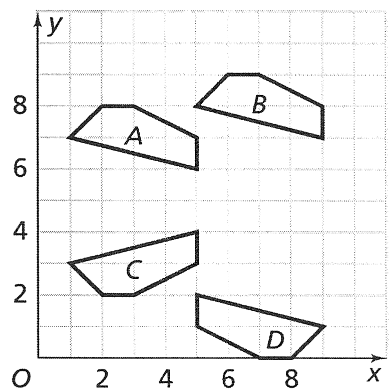
4. How can you expand the rule you wrote in Problem A to cover the reflection of a polygon across a line?

Exercises

1. Copy the figure on graph paper and graph its image after a reflection across the line for $x = 3$.



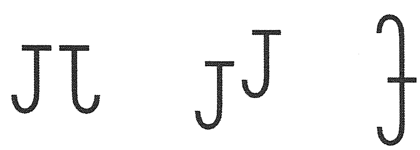
2. Evie was asked to draw three different reflections of figure A. Only one of her reflections is correct.



   a. Which figure is the reflection?

   b. What is the line of reflection?

3. Two of the pairs of letters represent a reflection.

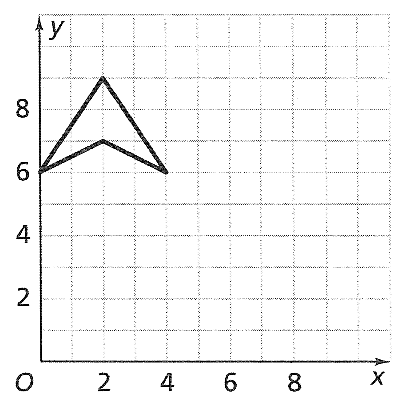


   a. Which pair does not represent a reflection?

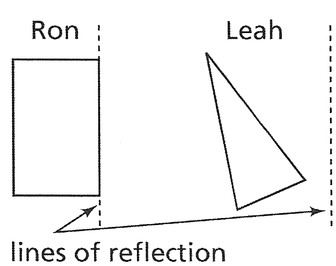
   b. Can any letter be flipped across a line of reflection?

   c. Flip your printed name over a line of reflection.

4. Tiara reflected the figure at the right and Deena translated it. Their new figures ended up in exactly the same location. Draw Tiara's reflected figure.



5. Ron and Leah wanted to show a reflection over a line by tracing a flat shape then flipping it over the line and tracing it again. Whose reflection will be more difficult to draw?



Topic 5: Reflections in the Coordinate Plane

At a Glance

PACING 1 day

Mathematical Goals

- Identify reflections used to move a polygon from one location to another in the coordinate plane.
- Explain how reflections affect the location of a polygon in the coordinate plane.

Guided Instruction

Once students understand the reflection of a point across a line, as presented in the first problem, the notion of reflecting a figure across a line by reflecting vertices and then connecting them should come fairly easily. Some students may have the mistaken impression that a line of reflection has to pass through at least one line of an original and reflected image so that the two images are touching. Problem B and Exercise 2 provide examples of reflections in which this is not the case.

The main emphasis in the lesson is the reflection of a geometric figure across a horizontal or vertical line in the coordinate plane. Although this lesson restricts itself to the first quadrant, you should use your judgment with regard to presenting examples of reflections across the x- and y-axis. If you do so, remind students to find reflected points simply by counting units between points and the line of reflection.

After Problem 5.1 A ask:

- *How do you think you could reflect a line segment across a line of reflection?* (Reflect the endpoints of the line segment and connect the two reflected points.)

After Problem 5.1 B ask:

- *Does the line of reflection have to be touching a figure and its reflected image?* (No)
- *How far away from a line of reflection can a figure and its reflected image be?* (There is no mathematical limit.)
- *What is the procedure for drawing the reflection of a triangle across a line that runs through the triangle?* (It is the same procedure as for a line of reflection exterior to the triangle: reflect the vertices across the line, then connect the reflected vertices.)

You will find additional work on transformations in the grade 8 unit *Kaleidoscopes, Hubcaps, and Mirrors.*

Vocabulary

- reflection
- line of reflection

Materials

- Labsheet 5ACE Exercises

ACE Assignment Guide for Topic 5

Core 1–5

Answers to Topic 5

Problem 5.1

A. 1. 2 units

2. *G'* is also 2 units from the line of reflection.

3. Yes

4. Answers may vary. Sample: To reflect a point across a line, plot a point on the opposite side of the line that is the same distance from the line as the original point.

B. 1. You would only see one triangle because the one triangle would be perfectly positioned over the other on.

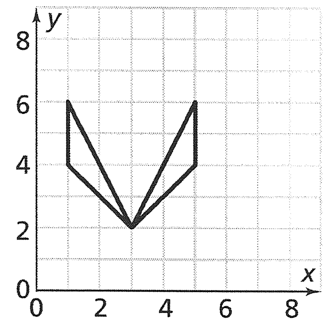
2. The two distances are the same.

3. Points *C* and *C'* are the same distance from the line, as are points *D* and *D'*.

4. Answers may vary. Sample: To reflect a polygon across a line, for each vertex plot a point on the opposite side of the line that is the same distance from the line as the original vertex. Connect the plotted points to form the reflected polygon.

Exercises

1.



2. a. *C*

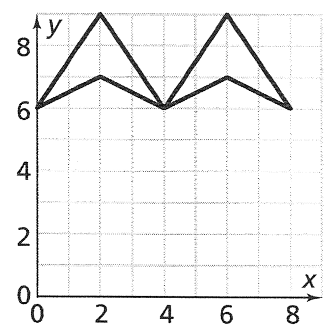
b. $y = 5$

3. a. (2)

b. yes

c. Check students' work.

4. Answers may vary. Sample:



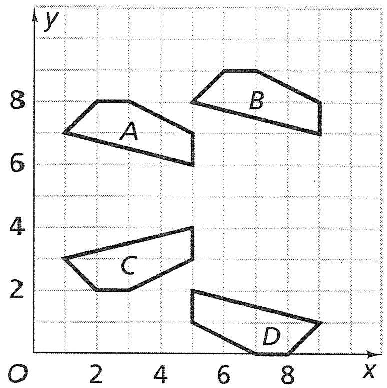
5. It will be more difficult for Leah because Ron's line of reflection is right up against a side of the rectangle, but Leah does not have that guidance, so the triangle could swivel when it is flipped and the vertices of the flipped image will not lie opposite the vertices of the original figure.

Name ______________________ Date __________ Class __________

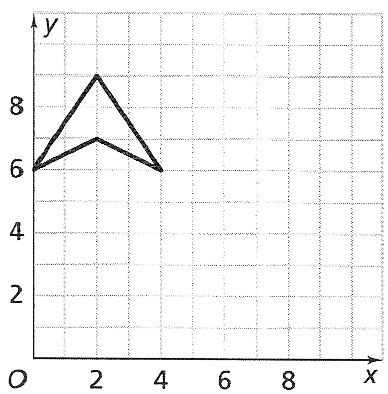
Labsheet 5ACE Exercises

Topic 5

1.



4.



Name ______________________________ Date __________ Class __________

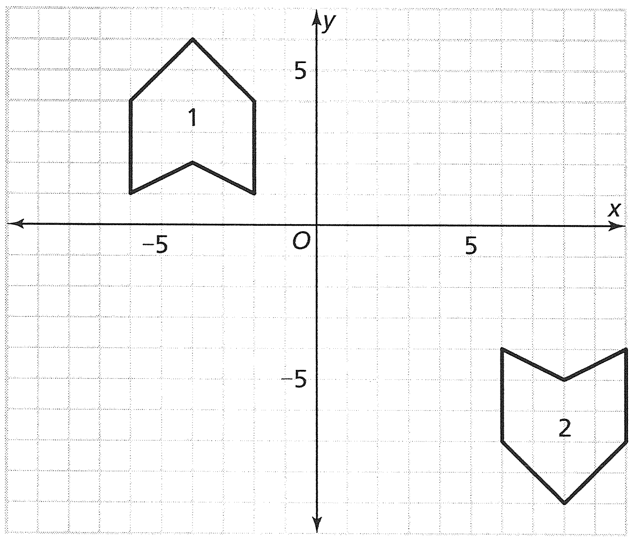
Additional Practice

Investigation 2

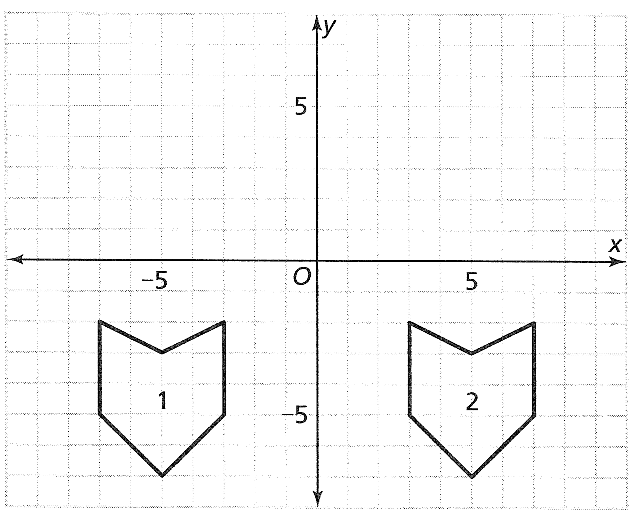
Kaleidoscopes, Hubcaps, and Mirrors

Describe a reflection or a combination of two reflections that would move Shape 1 to exactly match Shape 2.

1.



2.

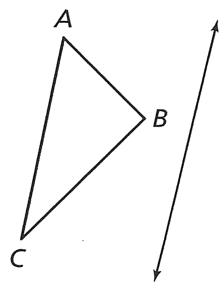


Name ______________________________ Date ___________ Class ___________

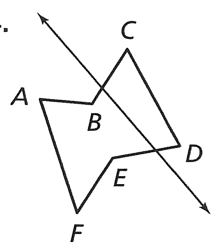
Additional Practice *(continued)*

Draw the image of the polygon under a reflection in the line. Describe what happens to each point on the original polygon under the reflection.

3.



4.

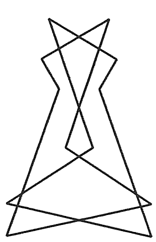


A shape and its image under a line reflection are given. Do parts (a) and (b).

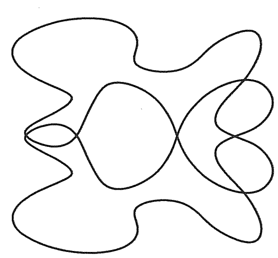
a. Draw the line of symmetry for the figure.

b. Label three points on the figure, and label the corresponding image points.

5.



6.



Name ______________________________ Date ____________ Class ____________

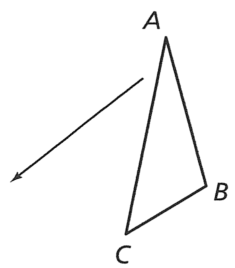
Additional Practice *(continued)*

Investigation 2

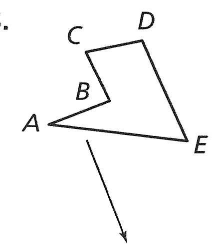
Kaleidoscopes, Hubcaps, and Mirrors

For Exercises 7 and 8, perform the translation indicated by the arrow. Describe what happens to each point of the original figure under the translation.

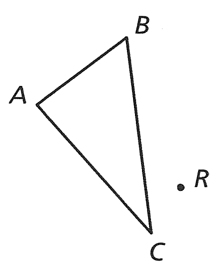
7.



8.



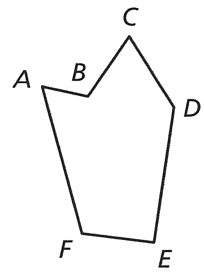
9. Rotate triangle ABC 90° *clockwise* about point R. Describe what happens to each point of triangle ABC under the rotation.



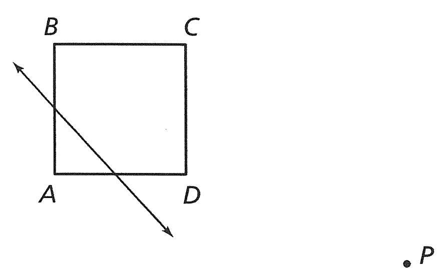
Name ______________________________ Date __________ Class __________

Additional Practice *(continued)*

10. Rotate polygon $ABCDEF$ 180° about point F. Describe what happens to each point of polygon $ABCDEF$ under the rotation.



For Exercises 11-13, refer to this diagram.



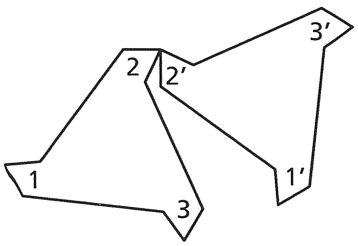
11. Draw the image of square $ABCD$ under a reflection in the line.

12. Draw the image of square $ABCD$ under a 45° rotation about point D.

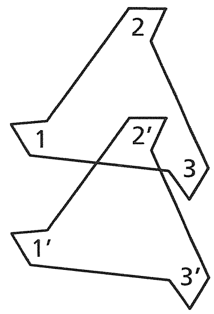
13. Draw the image of square $ABCD$ under the translation that slides point D to point P.

For Exercises 14–17, a polygon and its image under a transformation are given. Decide whether the transformation was a line reflection, a rotation, or a translation. Then indicate the reflection line, the center and angle of rotation, or the direction and distance of the translation.

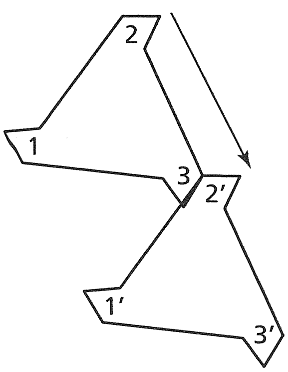
14.



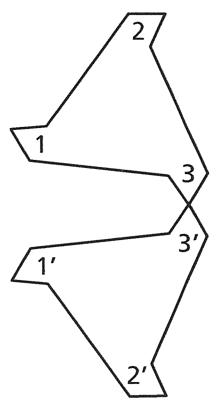
15.



16.



17.

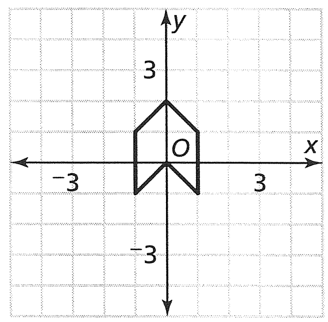


Name ____________________ Date ____________ Class ____________

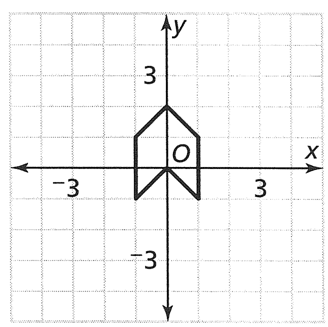
18. Suppose the shape below is translated according to the rolls of a six-sided number cube.

- If a 1, 2, or 3 is rolled, the shape is translated 3 units to the right.
- If a 4 is rolled, the shape is translated 3 units up.
- If a 5 is rolled, the shape is translated 3 units down.
- If a 6 is rolled, the shape is translated 3 units to the left

a. Draw the shape in its location after the following sequence of rolls: 3, 5, 6. What are the new coordinates of a general point (x, y) on the shape after this sequence of rolls?

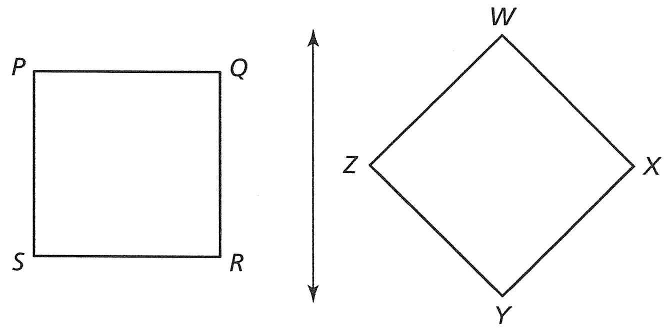


b. Draw the shape in its location after the following sequence of rolls: 1, 6, 4, What are the new coordinates of a general point (x, y) on the shape after this sequence of rolls?

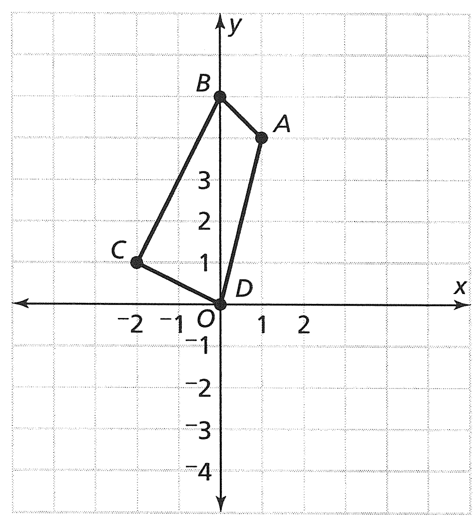


c. What sequence of rolls will produce a final image whose coordinates are all negative?

19. Describe two different sets of transformations that would move square *PQRS* onto square *WXYZ*.



20. Use the figure below to answer (a)–(g).



a. Write the coordinates of the points *A*, *B*, *C*, *D*.

b. Write the coordinates of the image of *ABCD* after a reflection in the *x*-axis.

c. Write the coordinates of the image of *ABCD* after a reflection in the *y*-axis.

d. Write the coordinates of the image of *ABCD* after a translation of 3 units to the right.

e. Write the coordinates of the image of *ABCD* after a translation of 4 units to the left.

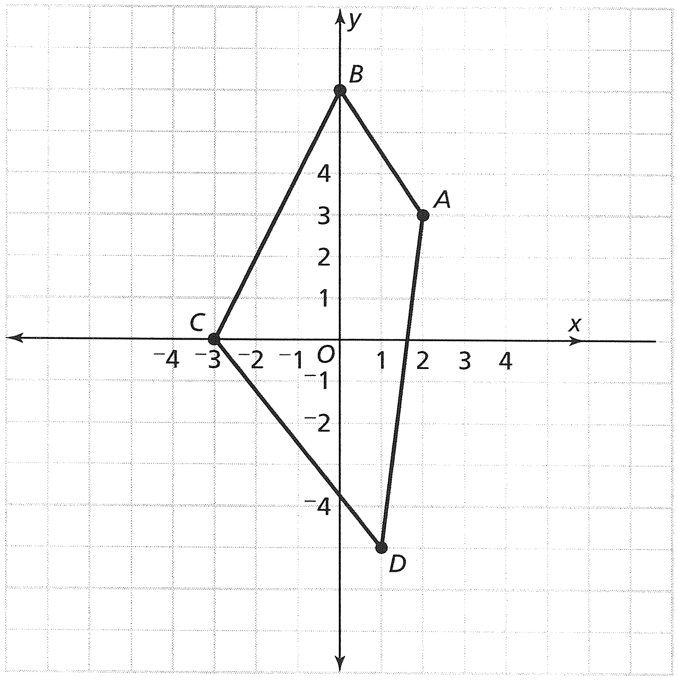
f. Write the coordinates of the image of *ABCD* after a translation of 2 units up.

g. Write the coordinates of the image of *ABCD* after a translation of 1 unit down.

Name ______________________________ Date ____________ Class ____________

Additional Practice *(continued)*

21. Use the figure below to answer parts (a)–(e).



a. Write the coordinates of the points A, B, C, D.

b. Write the coordinates of the image of $ABCD$ after a reflection in the line $x = 1$.

c. Write the coordinates of the image of $ABCD$ after a reflection in the line $x = -2$.

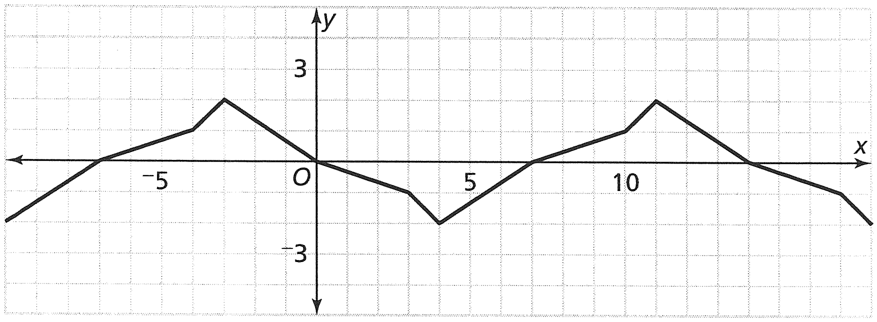
d. Write the coordinates of the image of $ABCD$ after a reflection in the line $y = 1$.

e. Write the coordinates of the image of $ABCD$ after a reflection in the line $y = -3$.

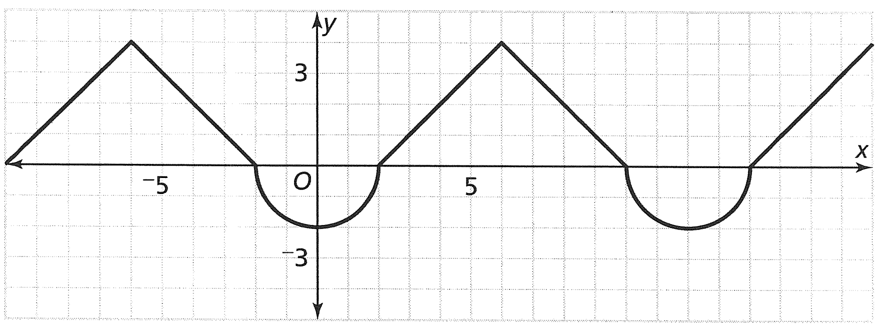
Additional Practice *(continued)*

For Exercises 22 and 23, suppose the pattern in the graph continues in both directions. Identify a basic design element that could be copied and transformed to make the entire pattern, and describe how the pattern could be made from that design element.

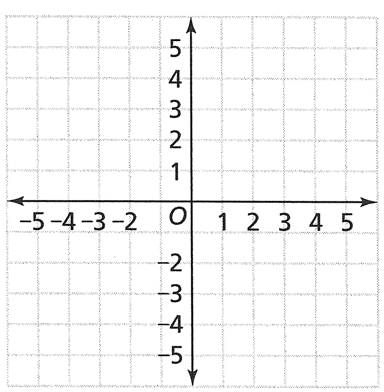
22.



23.



24. Plot the points (2, 4), (3, 5), (5, 5), (4, 4), (5, 3), and (3, 3) on a coordinate grid. Form a polygon by connecting the points in order and then connecting the last point to the first point. Reflect the polygon in the *y*-axis. Then translate the image 6 units to the right. Finally, rotate the second image 90° about the origin. What are the coordinates of the vertices of the final image?

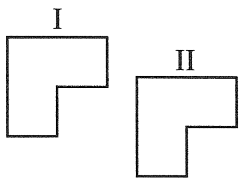


Name ______________________________ Date __________ Class __________

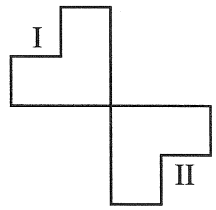
Skill: Analyzing Transformations

Figure II is the image of Figure I. Identify the transformation as a translation, a reflection, or a rotation.

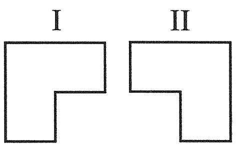
1.



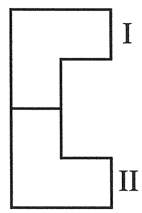
2.



3.

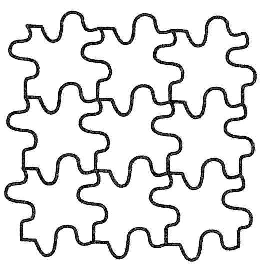


4.

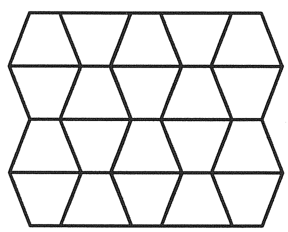


Describe the symmetries of each tessellation. Copy a portion of the tessellation, and draw any centers of rotational symmetry or lines of symmetry.

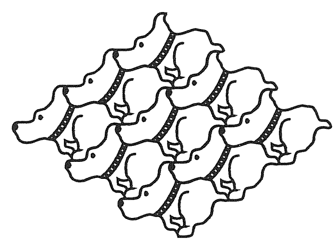
5.



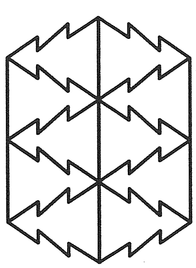
6.



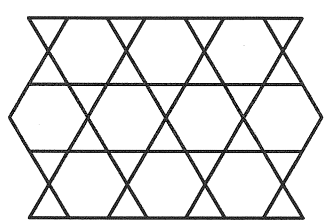
7.



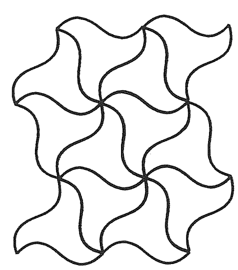
8.



9.



10.



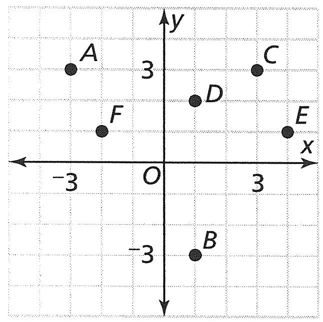
Name ______________________ Date __________ Class __________

Additional Practice

Investigation 5

Kaleidoscopes, Hubcaps, and Mirrors

For Exercises 1–6, refer to the grid below.



1. What are the coordinates of the image of point A under a translation that moves point $(1, 2)$ onto point $(-2, 0)$?

2. What are the coordinates of the image of point B under a translation that moves point $(1, 2)$ onto point $(4, -4)$?

3. What are the coordinates of the image of point C under a translation that moves point $(1, 2)$ onto point $(-3, -2)$?

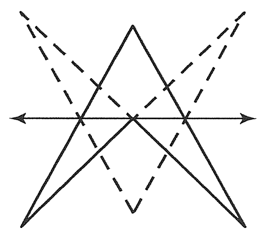
4. What are the coordinates of the image of point D under a reflection in the x-axis?

5. What are the coordinates of the image of point E under a reflection in the y-axis?

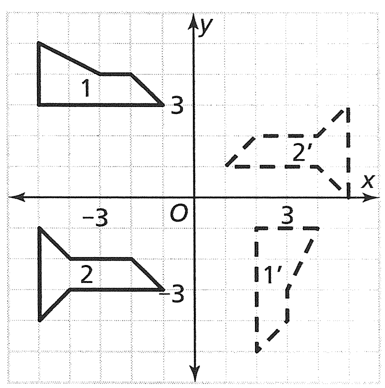
6. What are the coordinates of the image of point F under a reflection in the line $y = x$?

Additional Practice *(continued)*

7. Identify two congruent shapes in the figure below, and explain how you could use symmetry transformations to move one shape onto the other.



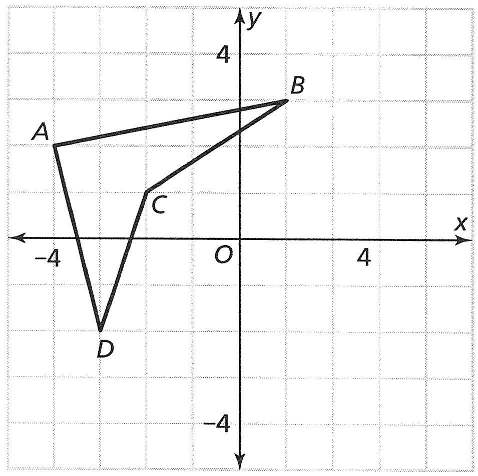
For Exercises 8–9, refer to the grid below.



8. Describe how you could move Shape 1 to exactly match Shape 1′ by using at least one translation and at least one reflection.

9. Describe how you could move Shape 2 to exactly match Shape 2′ by using at least one translation and at least one reflection.

For Exercises 10–12, refer to the grid below.



10. a. On the above grid, draw the final image created by rotating polygon $ABCD$ 90° counterclockwise about the origin and then reflecting the image in the x-axis.

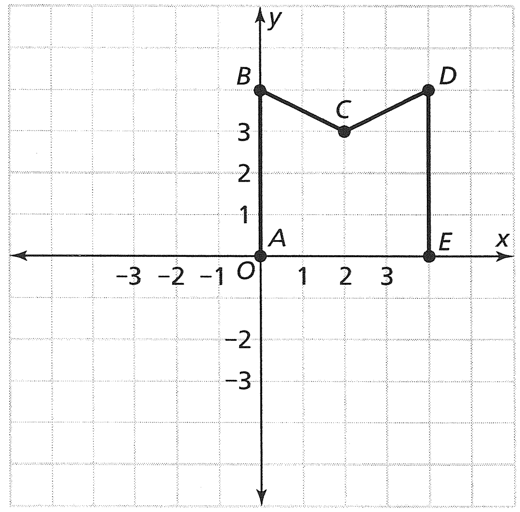
b. On the above grid, draw the final image created by reflecting polygon $ABCD$ in the x-axis and then rotating the image 90° counterclockwise about the origin.

c. Are the final images in parts (a) and (b) the same? Explain.

11. What single transformation is equivalent to a counterclockwise rotation of 90° about the origin followed by a rotation of 270° counterclockwise about the origin?

12. What single transformation is equivalent to a reflection in the y-axis, followed by a reflection in the y-axis, followed by a reflection in the y-axis?

13. Use the figure at the right to answer parts (a)–(c).

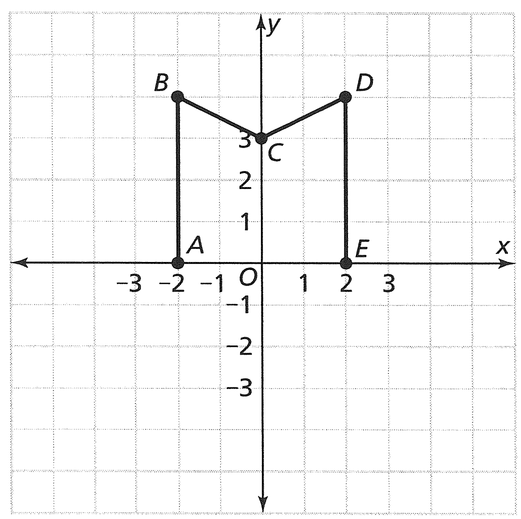


a. Write the coordinates for point A, B, C, D, E.

b. If the figure (the "M") was reflected in the x-axis, write the coordinates of the images of A, B, C, D and E.

c. If the figure (the "M") was reflected in the y-axis, write the coordinates of the images of A, B, C, D and E.

14. Use the figure at the right to answer parts (a)–(c).



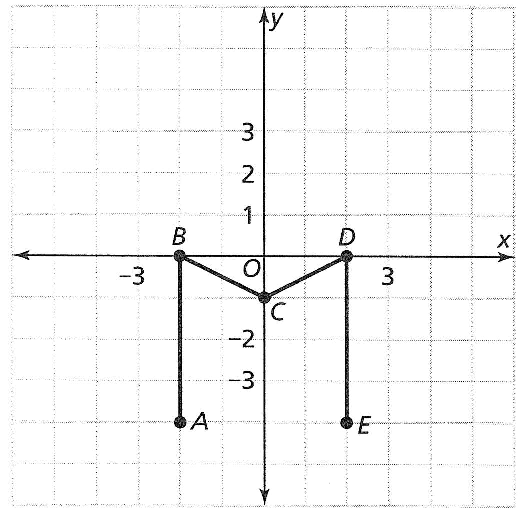
a. Write the coordinates for point A, B, C, D, E.

b. If the figure (the "M") was reflected in the x-axis, write the coordinates of the images of A, B, C, D and E.

c. If the figure (the "M") was reflected in the y-axis, write the coordinates of the images of A, B, C, D and E.

Name ______________________________ Date ____________ Class ____________

15. Use the figure at the right to answer parts (a)–(c).



a. Write the coordinates for point A, B, C, D, E.

b. If the figure (the "M") was reflected in the x-axis, write the coordinates of the images of A, B, C, D and E.

c. If the figure (the "M") was reflected in the y-axis, write the coordinates of the images of A, B, C, D and E.

16. Complete the table:

Point	Transformation	Coordinates of the Image
(3, 2)	Reflection in the x-axis	
(3, 1)	Reflection in the x-axis	
(3, 0)	Reflection in the x-axis	
(3, −1)	Reflection in the x-axis	
(3, −2)	Reflection in the x-axis	

17. Complete the table:

Point	Transformation	Coordinates of the Image
(3, 2)	Reflection in the y-axis	
(2, 2)	Reflection in the y-axis	
(1, 2)	Reflection in the y-axis	
(0, 2)	Reflection in the y-axis	
(−1, 2)	Reflection in the y-axis	

18. Describe the types of points, which are fixed after a reflection in the x-axis.

19. Describe the types of points, which are fixed after a reflection in the y-axis.

20. Complete the table:

Point	Transformation	Coordinates of the Image
(2, 4)	Reflection in the line $y = x$	
(2, 3)	Reflection in the line $y = x$	
(2, 2)	Reflection in the line $y = x$	
(2, 1)	Reflection in the line $y = x$	
(2, 0)	Reflection in the line $y = x$	

21. Complete the table:

Point	Transformation	Coordinates of the Image
(2, 4)	Reflection in the line $y = x$	
(3, −4)	Reflection in the line $y = x$	
(4, 4)	Reflection in the line $y = x$	
(−5, 4)	Reflection in the line $y = x$	
(−2, 0)	Reflection in the line $y = x$	

22. Describe the types of points, which are fixed after a reflection in the line $y = x$.

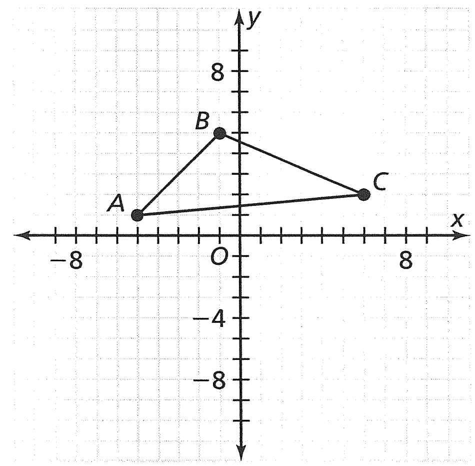
Name ______________________________ Date __________ Class __________

Skill: Transforming Coordinates

Investigation 5

Kaleidoscopes, Hubcaps, and Mirrors

$\Delta A'B'C'$ is a reflection of ΔABC over the x-axis. Draw $\Delta A'B'C'$ and complete each statement.

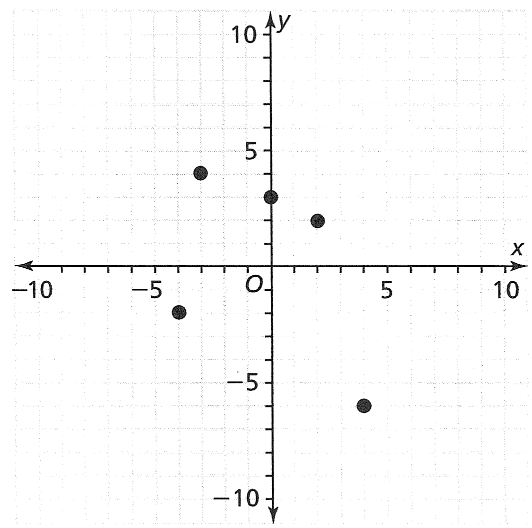


1. $A(-5,1) \rightarrow A'(x,y)$

2. $B(-1,5) \rightarrow B'(x,y)$

3. $C(6,2) \rightarrow C'(x,y)$

Graph each point and its reflection across the indicated axis. Write the coordinates of the reflected point.



4. $(-3, 4)$ across the y-axis

5. $(-4, -2)$ across the x-axis

6. $(2, 2)$ across the x-axis

7. $(0, 3)$ across the x-axis

8. $(4, -6)$ across the y-axis

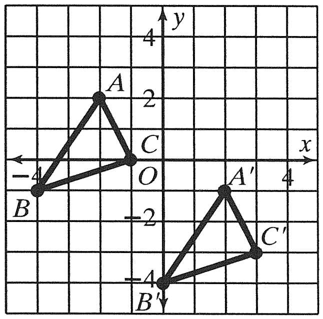
9. $(-4, -2)$ across the y-axis

Name ______________________________ Date __________ Class __________

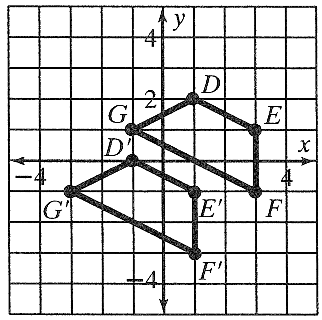
Skill: Transforming Coordinates *(continued)*

Write a rule to describe each translation.

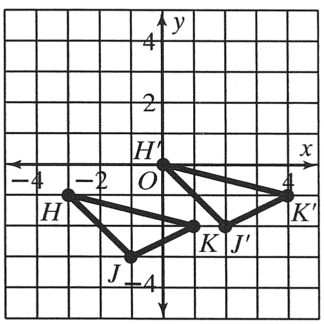
10. $(x, y) \rightarrow$



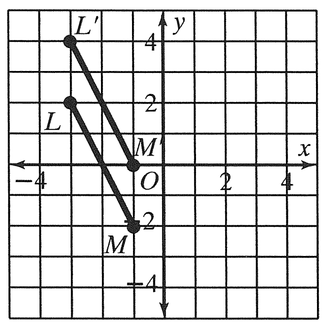
11. $(x, y) \rightarrow$



12. $(x, y) \rightarrow$



13. $(x, y) \rightarrow$



A point and its image after a translation are given. Write a rule to describe the translation.

14. $A(9, -4), A'(2, -1)$ $(x, y) \rightarrow$

15. $B(-3, 5), B'(-5, -3)$ $(x, y) \rightarrow$

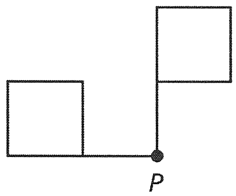
Write a rule to describe each statement.

16. In a 90° rotation, move point (x, y) to

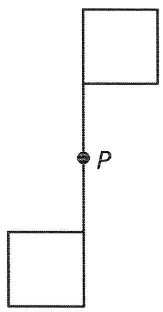
17. In a 180° rotation, move point (x, y) to

Kaleidoscopes, Hubcaps and Mirrors Answers

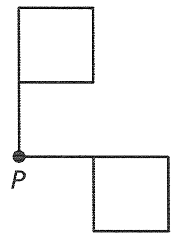
11. a.



b.



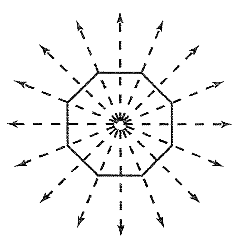
c.



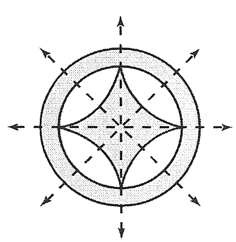
Skill: Identifying Reflection Symmetry

1. 1 **2.** 2 **3.** 2 **4.** 1

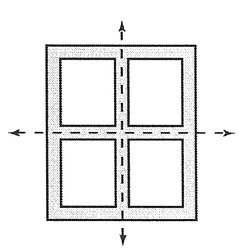
5.



6.



7.



8. no

9. yes **10.** yes

Skill: Identifying Rotation Symmetry

1. no **2.** yes; 180° **3.** yes; 90°
4. no **5.** yes, 90° **6.** no
7. yes, 270° **8.** yes, 315° **9.** yes, 360°

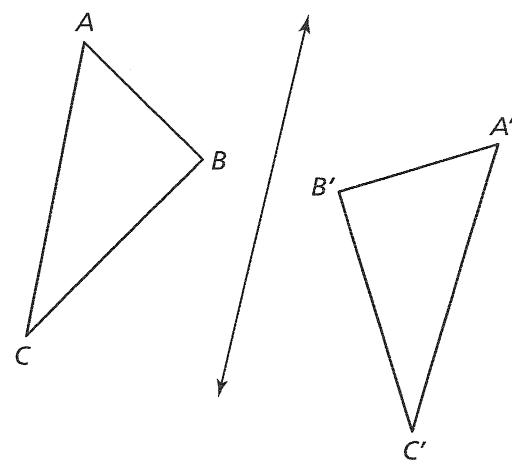
Investigation 2 Additional Practice

1. a reflection over the y-axis

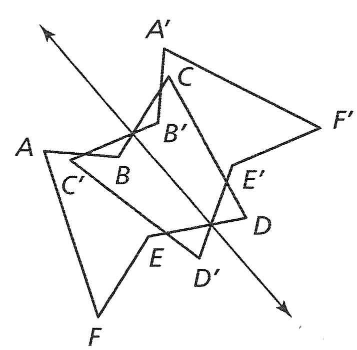
2. Possible answer: a reflection over the line $x = 2$, followed by a reflection over the line $y = -1.5$

In Exercises 3 and 4, each point is matched to an image point on the other side of the line of reflection. The image point lies on the line passing through the original point, perpendicular to the line of reflection. The distance from the image point to the line of reflection is equal to the distance from the original point to the line of reflection.

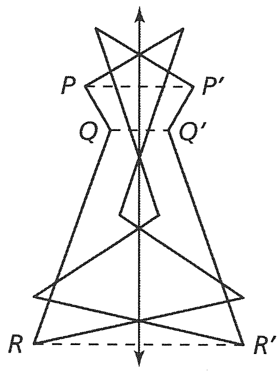
3.



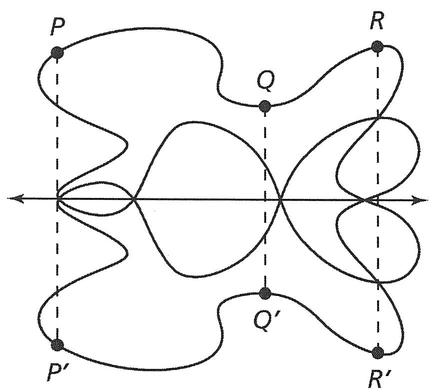
4.



5.

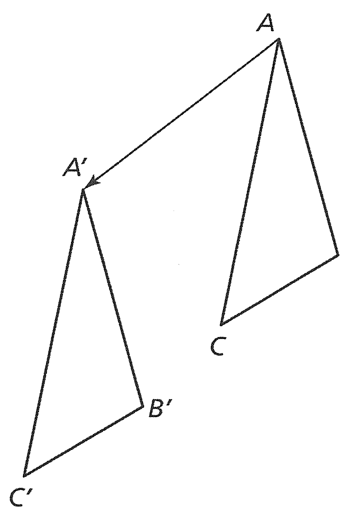


6.

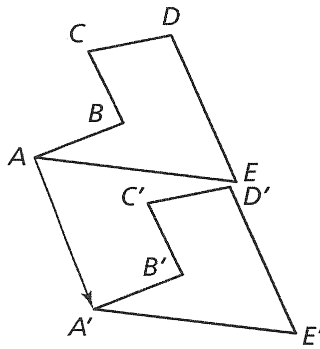


In Exercises 7 and 8, each point on the original figure is matched to an image point whose distance and direction from the original point are determined by the arrow.

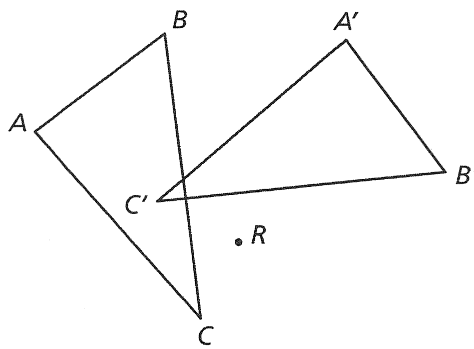
7.



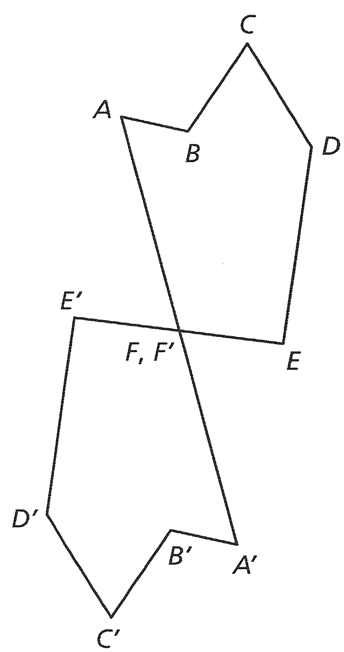
8.



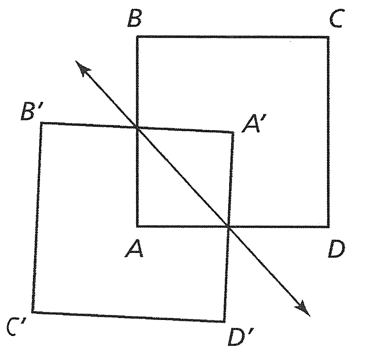
9. Each point X on triangle ABC is matched to an image point X' so that $\overline{RX} = \overline{RX'}$ and the measure of angle XRX' is 90°.



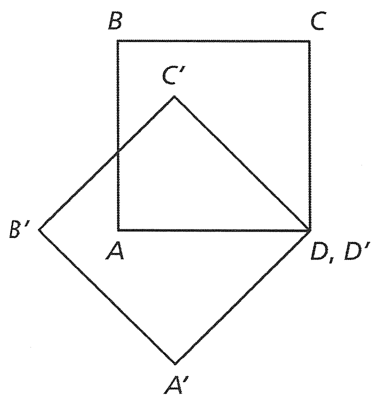
10. Each point X on polygon $ABCDEF$ is matched to an image point X' so that $\overline{FX} = \overline{FX'}$ and the measure of angle XFX' is 180°.



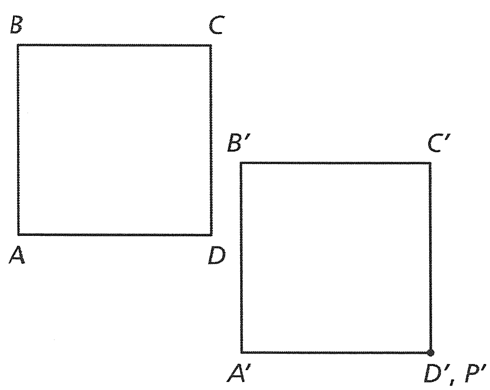
11.



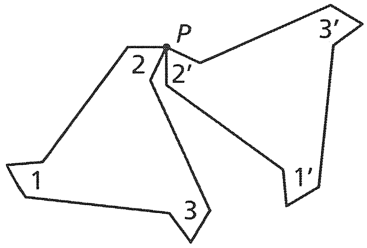
12.



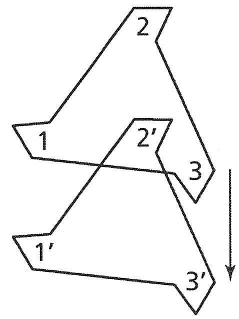
13.



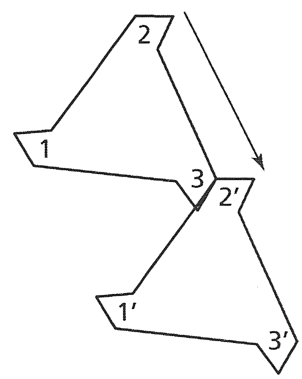
14. a rotation of 90° about point P



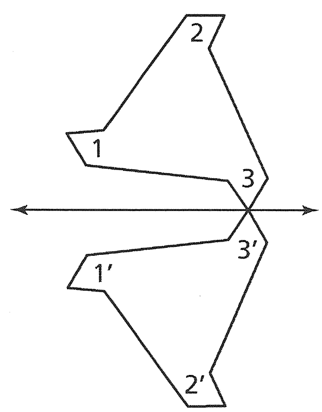
15. a translation with the length and direction indicated by the arrow



16. a translation with the length and direction indicated by the arrow

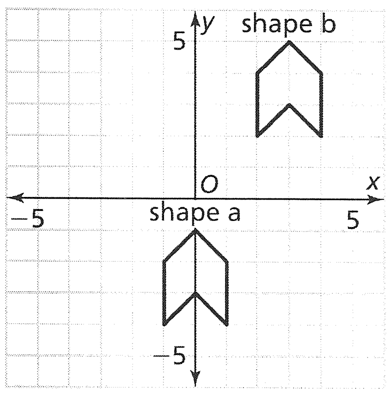


17. a reflection over the line shown



18. a. The result is labeled *shape a* in the drawing below. The coordinates of a general point (x, y) after the sequence of rolls is $(x, y - 3)$.

b. The result is labeled *shape b* in the drawing below. The coordinates of a general point (x, y) after the sequence of rolls is $(x + 3, y + 3)$.



c. Possible answer: 5, 6

19. Possible answer: Reflect square *PQRS* over the line and then rotate it 45° counterclockwise about its centerpoint. Rotate square *PQRS* 225° and then translate it until vertex *P* matches vertex *Y*.

20.

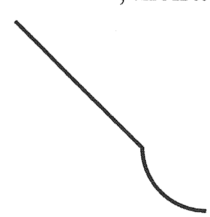
	A	B	C	D
a.	(1, 3)	(0, 4)	(−2, 1)	(0, 0)
b.	(1, −3)	(0, −4)	(−2, −1)	(0, 0)
c.	(−1, 3)	(0, 4)	(2, 1)	(0, 0)
d.	(4, 3)	(3, 4)	(1, 1)	(3, 0)
e.	(−3, 3)	(−4, 4)	(−6, 1)	(−4, 0)
f.	(1, 5)	(0, 6)	(−2, 3)	(0, 2)
g.	(1, 2)	(0, 3)	(−2, 0)	(0, −1)

21.

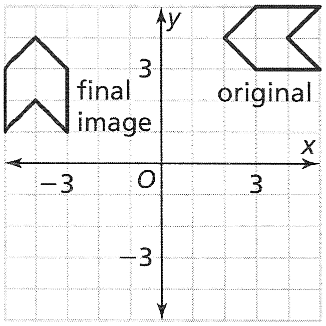
	A	B	C	D
a.	(2, 3)	(0, 6)	(−3, 0)	(1, −4)
b.	(0, 3)	(2, 6)	(5, 0)	(1, −4)
c.	(−6, 3)	(−4, 6)	(−1, 0)	(−5, −4)
d.	(2, −1)	(0, −4)	(−3, 2)	(1, 6)
e.	(2, −9)	(0, −12)	(−3, −6)	(1, −2)

22. One possible basic design element is shown below. You can generate the entire pattern by repeatedly translating this element 7 units to the right or left and then reflecting alternate elements over the x-axis.

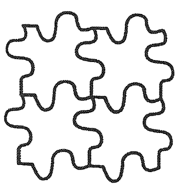
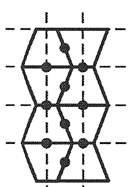
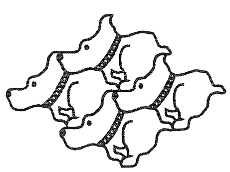
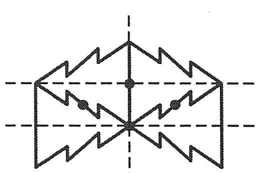
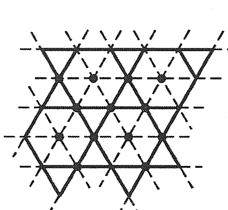
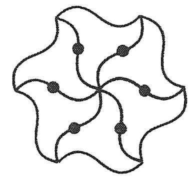
23. One possible basic design element is shown below. If you start with the copy of this element immediately to the left of the y-axis, you can generate the right half of the pattern by reflecting this element over the line $x = 0$, then $x = 6$, then $x = 12$, then $x = 18$, and so on. You can generate the left half of the pattern by reflecting this element over the line $x = -6$, then $x = -12$, then $x = -18$, and so on.



24. The coordinates of the final image are $(-4, 4), (-5, 3), (-5, 1), (-4, 2), (-3, 1)$, and $(-3, 3)$. Since $(x, y) \rightarrow (-x, y) \rightarrow (-x + 6, y) \rightarrow (-y, -x + 6)$.



Skill: Analyzing Transformations

1. translation
2. reflection or rotation
3. reflection or rotation
4. reflection or rotation
5. 
translational symmetry
6. 
line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
7. 
translational symmetry
8. 
line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
9. 
line symmetry, rotational symmetry, translational symmetry, glide reflectional symmetry
10. 
rotational symmetry, translational symmetry

Investigation 3 Additional Practice

1. a. $\overline{AB} \cong \overline{DF}$ $\quad \overline{BC} \cong \overline{FE}$ $\quad \overline{CA} \cong \overline{ED}$
$\angle A \cong \angle D$ $\quad \angle B \cong \angle F$ $\quad \angle C \cong \angle E$

b. $\overline{AB} \cong \overline{GH}$ $\quad \overline{BC} \cong \overline{HK}$ $\quad \overline{CA} \cong \overline{KG}$
$\angle A \cong \angle G$ $\quad \angle B \cong \angle H$ $\quad \angle C \cong \angle K$

c. $\overline{AB} \cong \overline{TS}$ $\quad \overline{BC} \cong \overline{SR}$ $\quad \overline{CA} \cong \overline{RT}$
$\angle A \cong \angle T$ $\quad \angle B \cong \angle S$ $\quad \angle C \cong \angle R$

2. a. $\overline{AD} \cong \overline{BC}$ correspond to $\overline{FE} \cong \overline{HG}$
$\angle A \cong \angle B$ correspond to $\angle F \cong \angle H$
$\angle C \cong \angle D$ correspond to $\angle G \cong \angle E$

b. $\overline{AD} \cong \overline{BC}$ correspond to $\overline{NM} \cong \overline{PR}$
$\overline{AB} \cong \overline{DC}$ correspond to $\overline{NP} \cong \overline{MR}$
$\angle A \cong \angle C$ correspond to $\angle N \cong \angle R$
$\angle B \cong \angle D$ correspond to $\angle P \cong \angle M$

3. a. $\triangle JNM \cong \triangle JNK \cong \triangle KNL \cong \triangle LNM$

b. $\triangle MLK \cong \triangle MKJ \cong \triangle JLK \cong \triangle JLM$

4. $\triangle ABD \cong \triangle CDB$; since $\overline{AD} \cong \overline{BC}$, $\overline{AB} \cong \overline{CD}$, and $\overline{DB} \cong \overline{BD}$ by SSS the triangles are congruent.

5. $\triangle ABC \cong \triangle CDA$; since $\overline{AD} \cong \overline{CB}$, $\overline{DC} \cong \overline{BA}$, and $\overline{CA} \cong \overline{AC}$ by SSS the triangles are congruent.

6. Using SSS $\cong$ SSS as was done in Exercises 4 and 5, the following pairs of triangles are congruent:
$\triangle ABD \cong \triangle CDB$ $\qquad \triangle ABC \cong \triangle CDA$
$\triangle ABN \cong \triangle CDN$ $\qquad \triangle ADN \cong \triangle CBN$

7. a. $A(-2, -1); B(2, -1); C(2, 2)$
b. $A'(1, -2); B'(1, -2); C'(-2, 2)$
c. Rotations preserve the shape of the original triangle so the triangle and its image are congruent.

Investigation 4 Additional Practice

1. a. Yes; by SSS $\cong$ SSS since $\overline{AC} \cong \overline{AC}$, $\overline{AD} \cong \overline{AB}$, and $\overline{DC} \cong \overline{BC}$.

   b. Yes; Corresponding parts of congruent triangles are congruent so angles DAC and BAC are congruent. Thus $\overline{AC}$ bisects angle DAB.

   c. Yes; Corresponding parts of congruent triangles are congruent so angles DCA and BCA are congruent. Thus $\overline{AC}$ bisects angle DCB.

2. a. Yes; by SSS $\cong$ SSS since $\overline{AD} \cong \overline{AD}$, $\overline{AC} \cong \overline{AB}$ (since triangle ABC is isosceles) and $\angle ADC \cong \angle ADB$ are right angles (since $\overline{BC}$ is a straight angle).

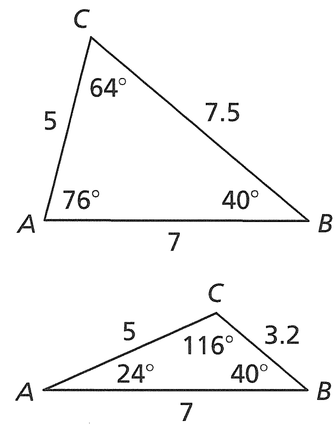
   b. Yes; Corresponding parts of congruent triangles are congruent so angles CAD and BAD are congruent. Thus $\overline{AD}$ bisects angle CAB.

3. a. $\overline{AP} \cong \overline{BP}, \overline{CP}, \overline{DP}, \overline{EP}, \overline{FP}, \overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}$ and $\overline{EF}$; since the hexagon has 60° rotation symmetry and segments $\overline{AP}, \overline{BP}, \overline{CP}, \overline{DP}, \overline{EP}$ and $\overline{FP}$ form a design inside the hexagon which also has 60° degree rotation symmetry, the segments are congruent. Since the hexagon with the segments $\overline{AP}, \overline{BP}, \overline{CP}, \overline{DP}, \overline{EP}$ and $\overline{FP}$ drawn from point P is composed of 6 equilateral triangles $ABP, BPC, CPD, DPE, EPF, FPA$, we also get that $\overline{AP} \cong \overline{AB}, \overline{BC}, \overline{CD}, \overline{DE}, \overline{EF}$.

   b. $\angle PAB$ is congruent to every acute angle (for a total of 18 congruent angles) in the diagram, since all 6 triangles formed are the angles of an equilateral triangle and thus have a measure of 60°.

4. a. Yes; Given the information only one triangle is possible by SAS.

   b. No; Given SSA does not guarantee that you have a congruent triangle. For example, two triangles are drawn below each with sides $\overline{AB} = 5$ cm, $\overline{AC} = 7$ cm, $\angle B = 40°$; however, they are not congruent.

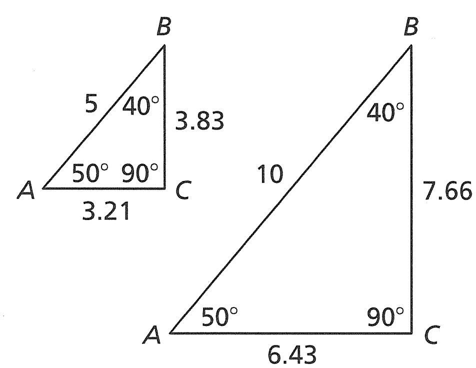


   c. Yes; Given the information only one triangle is possible by ASA.

   d. Yes; Given the information only one triangle is possible by ASA since the third angle is 50°.

   e. Yes; Given the information only one triangle is possible by SSS.

   f. No; Given AAA does not guarantee that you have a congruent triangle. For example, two triangles are drawn below each with $\angle A = 50°$, $\angle B = 40°$, $\angle C = 90°$ however they are not congruent.

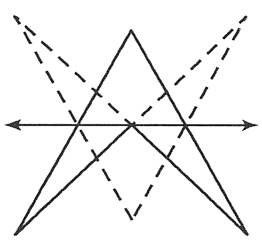


Skill: Using Congruence

1. $\triangle NLM$ 2. $\triangle FED$ 3. $\triangle RTS$
4. a. $\angle FED$ b. $\overline{FE}$ c. $\angle A$
5. a. $\triangle FED$ b. $\triangle EFD$ c. $\triangle DFE$
6. yes; $\triangle JKL \cong \triangle VUT$; SSS
7. no; not enough information given.
8. yes; $\triangle ABC \cong \triangle EDC$; ASA
9. yes; $\triangle WXY \cong \triangle ZYX$; SAS
10. yes 11. yes 12. no

Investigation 5 Additional Practice

1. $(-6, 1)$ 2. $(4, -9)$ 3. $(-1, -1)$
4. $(1, -2)$ 5. $(-4, 1)$ 6. $(1, -2)$
7. Possible answer: One of the congruent shapes is drawn with a dashed line and the other with a solid line. You can move one shape onto the other by reflecting it over the line shown.



8. Possible answer: Translate shape 1 down 1 unit and to the left 4 units. Then reflect it over the line $x = -5$. Finally, reflect it over the line $y = x$.
9. Possible answer: Reflect shape 2 over the y-axis and then translate it up 4 units.

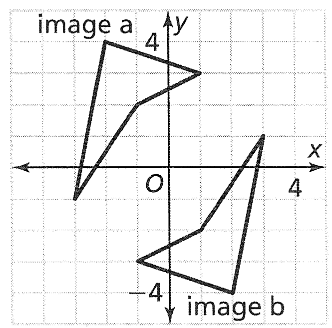
Figure 1

	A	B	C	D	E
a.	(−2, 0)	(−2, 4)	(0, 3)	(2, 4)	(2, 0)
b.	(−2, 0)	(−2, −4)	(0, −3)	(2, −4)	(2, 0)
c.	(2, 0)	(2, 4)	(0, 3)	(−2, 4)	(−2, 0)

Figure 2

	A	B	C	D	E
a.	(−2, −4)	(−2, 0)	(0, −1)	(2, 0)	(2, −4)
b.	(−2, 4)	(−2, 0)	(0, 1)	(2, 0)	(2, 4)
c.	(2, −4)	(2, 0)	(0, −1)	(−2, 0)	(−2, −4)

10. a. The final image is labeled image a.
b. The final image is labeled image b.
c. The images are not the same. Rotating a figure 90° counterclockwise about the origin and then reflecting it over the x-axis takes point (x, y) to $(-y, x)$ and then to $(-y, -x)$. Reflecting a figure over the x-axis and then rotating it 90° counterclockwise about the origin takes point (x, y) to $(x, -y)$ and then to (y, x).



11. a 360° rotation about the origin
12. a reflection over the x-axis
13.

	A	B	C	D	E
a.	(0, 0)	(0, 4)	(2, 3)	(4, 4)	(4, 0)
b.	(0, 0)	(0, −4)	(2, −3)	(4, −4)	(4, 0)
c.	(0, 0)	(0, 4)	(−2, 3)	(−4, 4)	(−4, 0)

14. (Figure 1)
15. (Figure 2)

16.

Point	Transformation	Coordinates of the Image
(3, 2)	Reflection in the x-axis	(3, −2)
(3, 1)	Reflection in the x-axis	(3, −1)
(3, 0)	Reflection in the x-axis	(3, 0)
(3, −1)	Reflection in the x-axis	(3, 1)
(3, −2)	Reflection in the x-axis	(3, 2)

17.

Point	Transformation	Coordinates of the Image
(3, 2)	Reflection in the y-axis	(−3, 2)
(2, 2)	Reflection in the y-axis	(−2, 2)
(1, 2)	Reflection in the y-axis	(−1, 2)
(0, 2)	Reflection in the y-axis	(0, 2)
(−1, 2)	Reflection in the y-axis	(1, 2)

18. All points on the x-axis are fixed. Thus all points of the form $(a, 0)$ where a is a number.

19. All points on the y-axis are fixed. Thus all points of the form $(0, b)$ where b is a number.

20.

Point	Transformation	Coordinates of the Image
(2, 4)	Reflection in the line $y = x$	(4, 2)
(2, 3)	Reflection in the line $y = x$	(3, 2)
(2, 2)	Reflection in the line $y = x$	(2, 2)
(2, 1)	Reflection in the line $y = x$	(1, 2)
(2, 0)	Reflection in the line $y = x$	(0, 2)

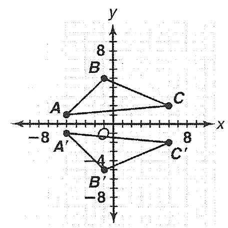
21.

Point	Transformation	Coordinates of the Image
(2, 4)	Reflection in the line $y = x$	(4, 2)
(3, −4)	Reflection in the line $y = x$	(−4, 3)
(4, 4)	Reflection in the line $y = x$	(4, 4)
(−5, 4)	Reflection in the line $y = x$	(4, −5)
(−2, 0)	Reflection in the line $y = x$	(0, −2)

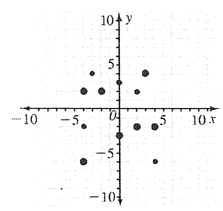
22. All points that lie on the line $y = x$ are fixed. Thus all points of the form (x, x) where the x coordinate and the y coordinate are equal.

Skill: Transforming Coordinates

1. $-5, -1$ **2.** $-1, -5$ **3.** $6, -2$



4. $(3, 4)$ **5.** $(-4, 2)$ **6.** $(2, -2)$



7. $(0, -3)$ **8.** $(-4, -6)$ **9.** $(4, -2)$

10. $(x + 4, y - 3)$ **11.** $(x - 2, y - 2)$

12. $(x + 3, y + 1)$ **13.** $(x, y + 2)$

14 $(x - 7, y + 3)$ **15.** $(x - 2, y - 8)$

16. $(-y, x)$ **17.** $(-x, -y)$

Reflection of Images

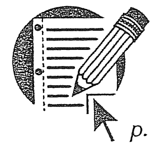
Goals

To assess students'—

- understanding that reflections preserve the properties of figures;
- ability to draw reflections of figures.

Materials and Equipment

- A copy of the blackline master "Reflection of Images" for each student
- Rulers
- Tracing paper



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Activity

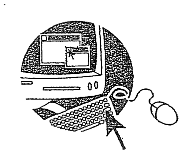
Students draw the image of each of four figures after the figures have been reflected over the y-axis of a coordinate grid, and then they draw the image of each figure after it has been reflected over the x-axis.

Discussion

The students might verify their answers by tracing the figure and then turning the paper over to show the reflected image of the original. If they say that the figures are the "same," ask them to explain in what ways the figures are the same. They should mention the lengths of the sides and the measures of the angles. Next, ask them why these properties remain unchanged. They should recognize that a reflection merely reorients an object without changing anything else about it. If the students fail to notice that the shape and size have not changed, prompt them to reflect the figure over one of the axes and observe the results. Once again, focus the students' attention on the figure's side lengths and angles. The degree to which students are troubled by the orientation of a figure can help you distinguish students with limited prior experiences from those who are familiar with transformations. For students with some background in transformations, orientation will be less troubling, and they should be able to use appropriate vocabulary to describe transformations.

Selected Instructional Activities

Tracing tasks give students experience in determining the image of a figure under a transformation. Exploring relationships between the preimage (original figure) and the image in translations, reflections, and rotations is the goal of the following activity (adapted from the Reconceptualizing Mathematics Project n.d.). The students may be more familiar with the terms *slide*, *flip*, and *turn* than with *translation*, *reflection*, and *rotation*. Allow the students to use the familiar terms to describe their actions, but also introduce the geometric vocabulary for such motions.



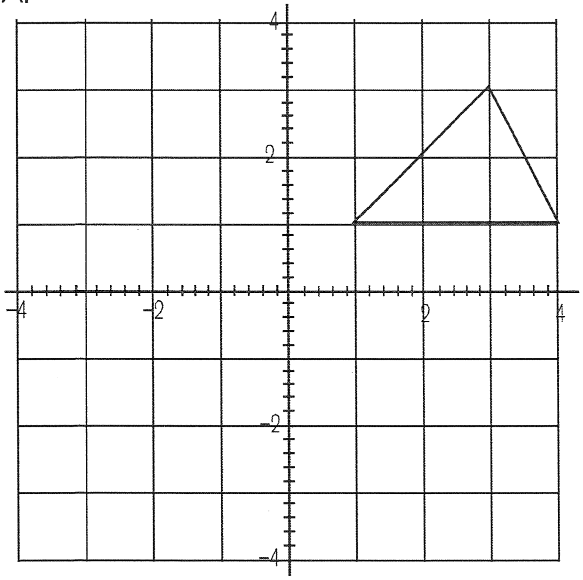
The CD-ROM includes an interactive applet, Transformation Tools, that can help students visualize transformations.

Reflection of Images

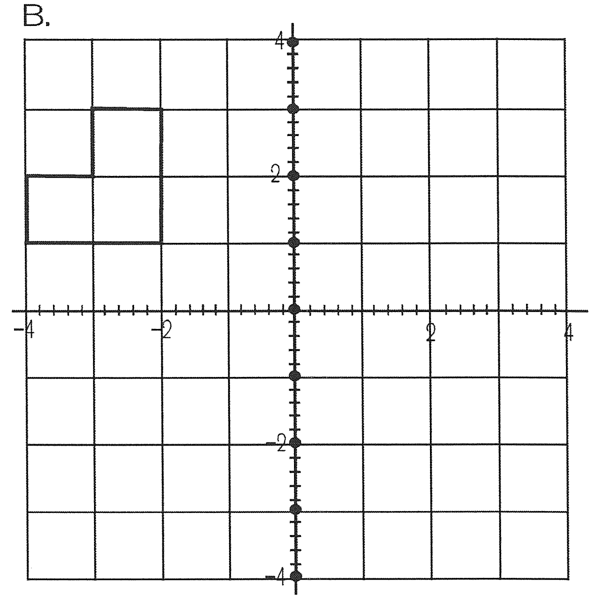
Name ___________________________

1. Draw the image of each of the four figures below when the original shape has been reflected across the y-axis.

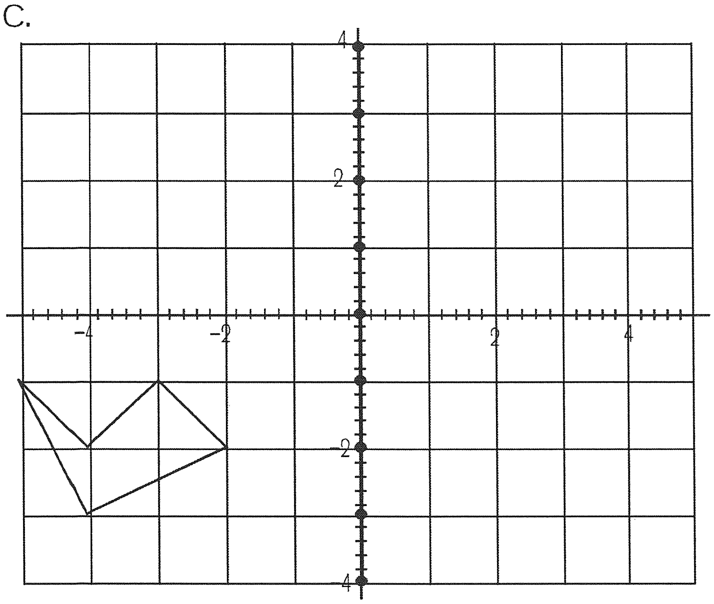
A.



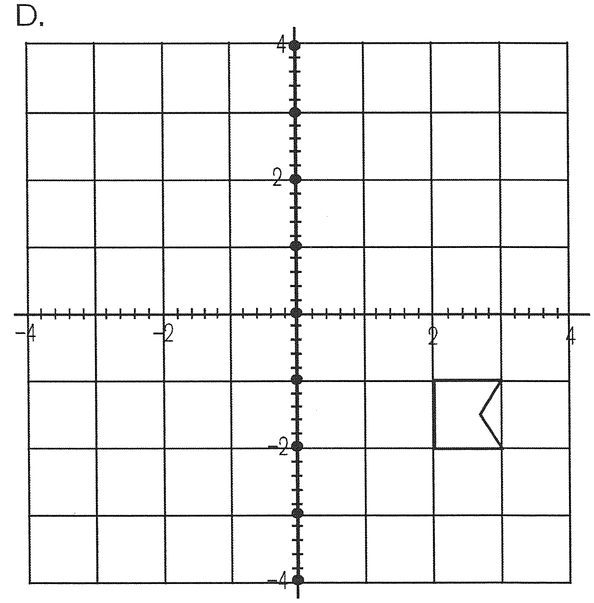
B.



C.



D.



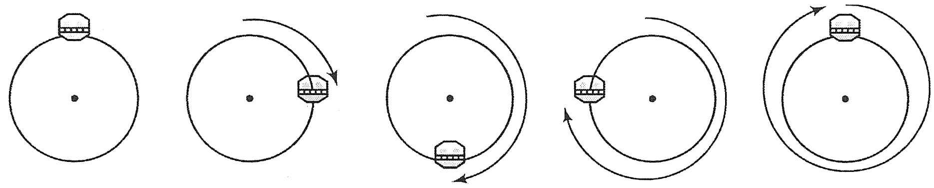
2. What image results when each of the original figures has been reflected across the x-axis? Draw the reflected figures.

Topic 6: Rotations in the Coordinate Plane

for use after ***Shapes and Designs*** **Investigation 4**

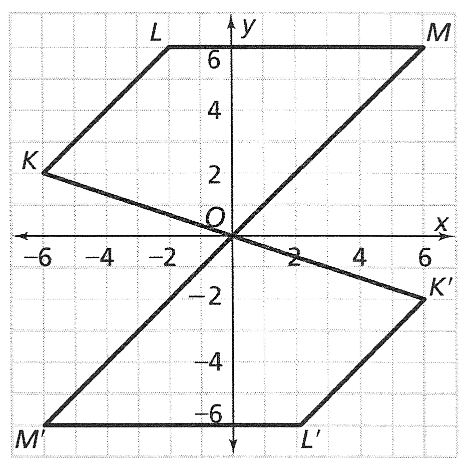
A **rotation** is a transformation that revolves a figure around a fixed point called the **center of rotation**. A rotation is **clockwise** if its direction is the same as that of a clock hand. A rotation in the other direction is called **counterclockwise**. A complete rotation is 360°.

A ferris wheel makes a 90° rotation with every $\frac{1}{4}$ turn.



Problem 6.1

The rotation of figure $KLMO$ 180° about (0, 0), which is called the origin of the coordinate plane, is shown. In $K'L'MO$, point K' (kay-prime) is the rotation of point K, point L' is the rotation of point L, and point M' is the rotation of point M.



A. 1. Use a ruler to compare the lengths of OM and OM'.

2. What other pairs of segments can you find that have the same length?

3. When a point is rotated, how does its distance to the center of rotation change?

4. Describe the movement of the point at the center of rotation.

5. When you rotate a figure 180°, does it matter whether you rotate clockwise or counterclockwise? Explain.

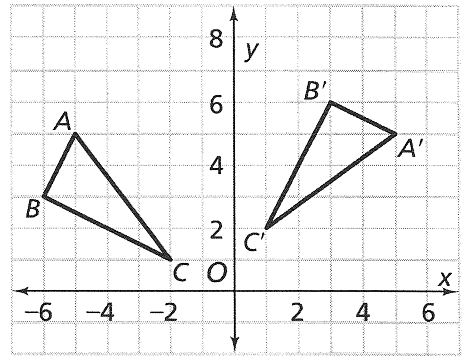
B. Describe how to rotate a polygon by using the locations of its vertices.

C. What is the new location of the point at (0, 6) after it has been rotated clockwise 180° about the origin?

D. If you think of (0, 6) and point L rotating together, how can that help you understand the position of L'?

Problem 6.2

The clockwise rotation of ΔABC 90°about the origin is shown at the right.



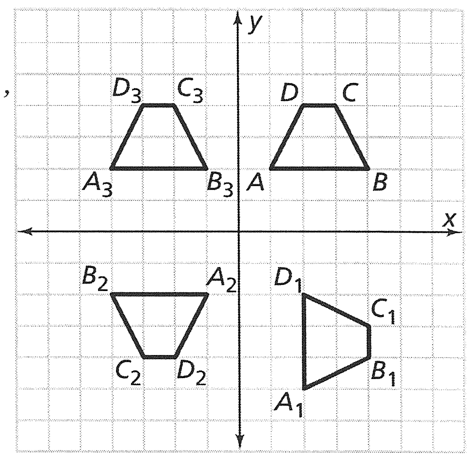
A. Compare the distances from the origin to points C and C'.

B. When a figure is rotated, does a vertex have to be the center of rotation?

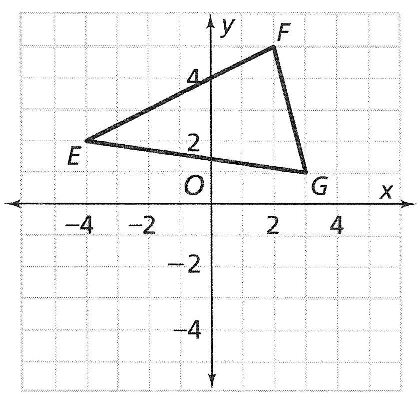
C. If you draw a line from A and a line from A' through the center of rotation, what is the measure of the angle formed at the intersection of the lines?

Exercises

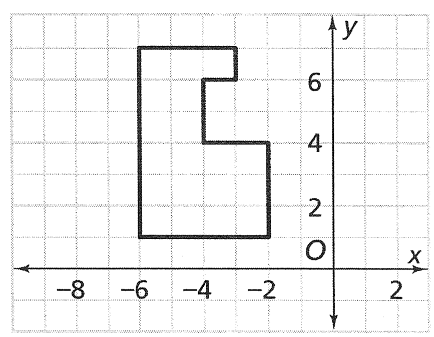
1. In the diagram, which figure is a rotation of figure $ABCD$? Explain how you know.



2. Copy ΔEFG onto graph paper and draw $\Delta E'F'G'$ as its image after a clockwise rotation of 180° about the origin.



3. Copy the figure at the right onto graph paper and draw its image after a counterclockwise rotation of 90° about the origin.



4. a. Explain how a 180° rotation of a shape tile can have the same visual results as a reflection.

b. How can you tell when a 180° rotation will have a different visual result?

Topic 6: Rotations in the Coordinate Plane

At a Glance

PACING 1 day

Mathematical Goals

- Identify rotations used to move a polygon from one location to another in the coordinate plane.
- Explain how rotations affect the location of a polygon in the coordinate plane.

Guided Instruction

Vocabulary

- rotation
- center of rotation
- clockwise
- counterclockwise

Materials

- Labsheets 6.1, 6.2, and 6ACE Exercises

Rotations can be more problematic for students than translations or reflections. Even the 180° rotation of a shape about the origin, as presented in Problem 6.1 can present difficulties as compared with a reflection of the same image across the *x*-axis. It may help students to concentrate on one vertex as a "leading point," visualize the arc made by that point, and then concentrate on the "trailing points" that follow the leader along the arcs of concentric circles.

No matter where the center of rotation is located—inside, outside or on the figure—the key indicator of a rotation is the distance from the center of rotation to any particular point on the figure. To confirm that a rotation has occurred, students can compare pre- and post-rotation distances of a given point with a compass, a ruler, or the marked edge of a piece of paper. They can also consider these two distances as diagonals of congruent rectangles formed on the coordinate grid. For instance, in Problem 6.2, a line segment from the origin to *C* or the origin to *C'* would both be the diagonal of a 2 unit × 1 unit rectangle and be of equal length.

Before Problem 6.1:

- *How many degrees of a counterclockwise rotation has the same effect as a 270° clockwise rotation?* (90°)
- *What part of the Ferris wheel at the top of the page does not change its location no matter what type of rotation takes place?* (the center)

After Problem 6.1:

- *How can you tell that K'L'M'O is not a reflection of KLMO across the x-axis?* (Corresponding points are not the same distance away from the x-axis on the opposite side.)
- *How would the graph look different if the 180° rotation had been counterclockwise instead of clockwise?* (There would be no difference.)

Before Problem 6.2:

- *Why do the two figures seem to be floating in space?* (The center of rotation is outside the figure.)
- *If you extend all three sides of both triangles, what type of angle do you think there would be where any the of the lines from corresponding sides intersect?* (90°)

You will find additional work on transformations in the grade 8 unit *Kaleidoscopes, Hubcaps, and Mirrors*.

ACE Assignment Guide for Topic 6

Core 1–5

Answers to Topic 6

Problem 6.1

A. 1. The lengths are the same.

2. $KO = K'O$, $KL = K'L'$, $LM = L'M'$

3. The distance remains the same.

4. There is no change in the position of the center of rotation.

5. No, it does not matter. They result in the same transformation.

B. Rotate each of the vertices, then draw the sides.

C. $(0, -6)$

D. Answers may vary. Sample: Since L stays 2 units behind $(0, 6)$ during the rotation, it ends up 2 units to the right of $(0, -6)$ after the rotation.

Problem 6.2

A. The distances are the same.

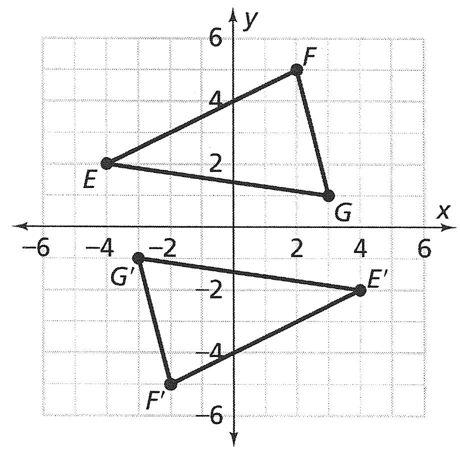
B. No

C. 90°

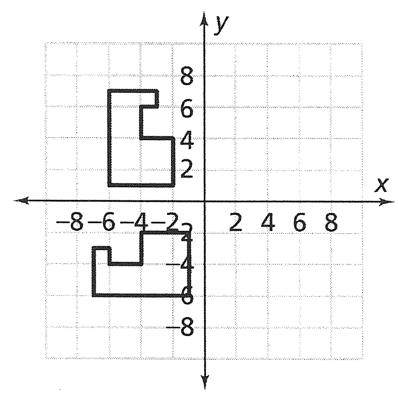
Exercises

1. Answers may vary. Sample: $A_2B_2C_2D_2$, because the distances from the origin to the pairs of corresponding vertices is the same.

2.



3.



4. a. Answers may vary. Sample: The tile has to be symmetrical about a vertical axis.

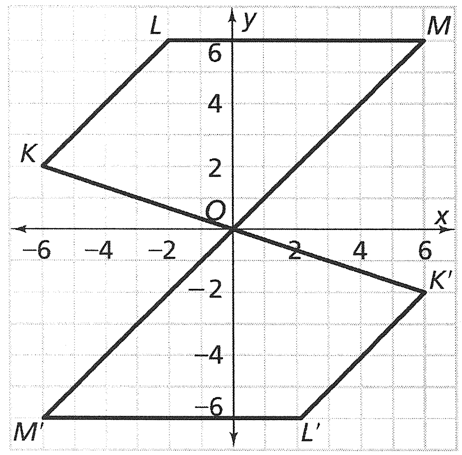
b. Answers may vary. Sample: The tile must not be symmetrical about a vertical axis.

Name ______________________ Date __________ Class __________

Labsheet 6.1

Topic 6

Rotations in the Coordinate Plane

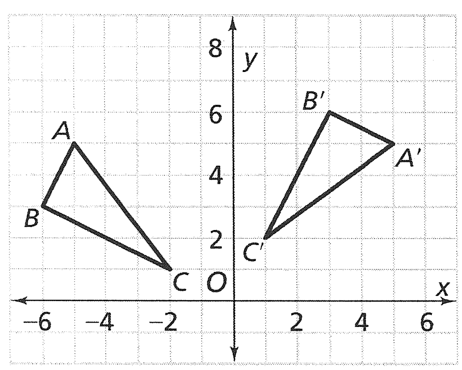


Name ______________________ Date __________ Class __________

Labsheet 6.2

Topic 6

Rotations in the Coordinate Plane

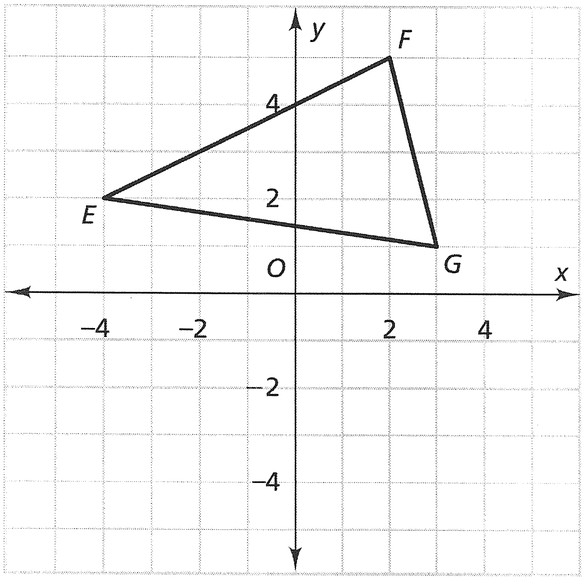


Name ______________________ Date __________ Class __________

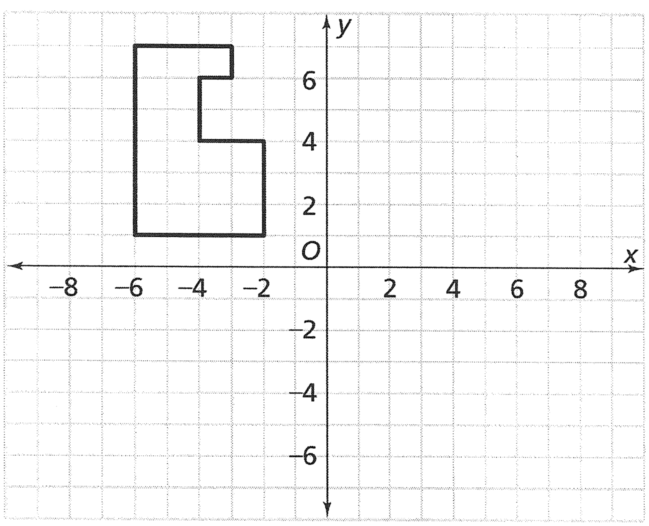
Labsheet 6ACE Exercises

Topic 6

2.



3.



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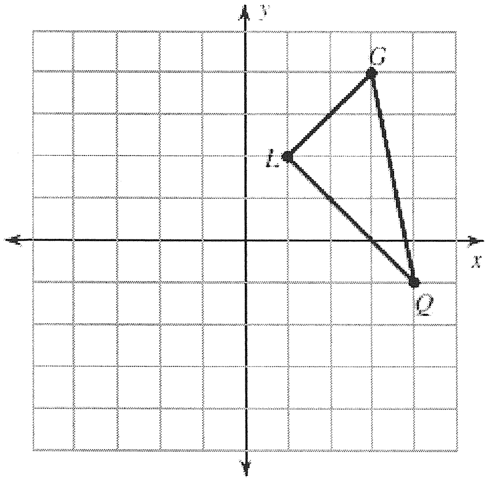
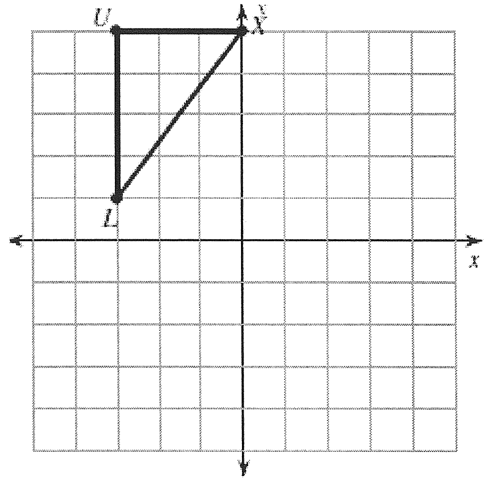
Practice with Reflections & Rotations

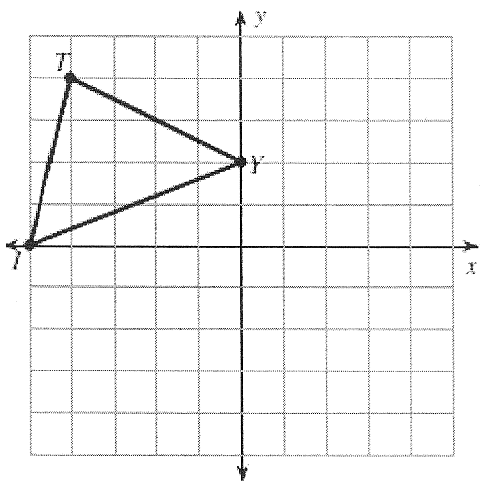
reflection across the x-axis



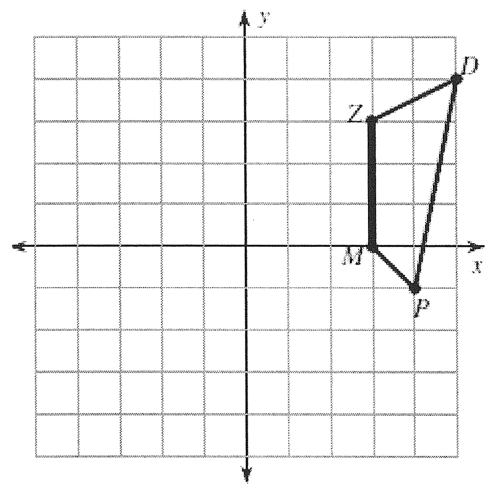
reflection across $y = 3$



reflection across $y = 1$

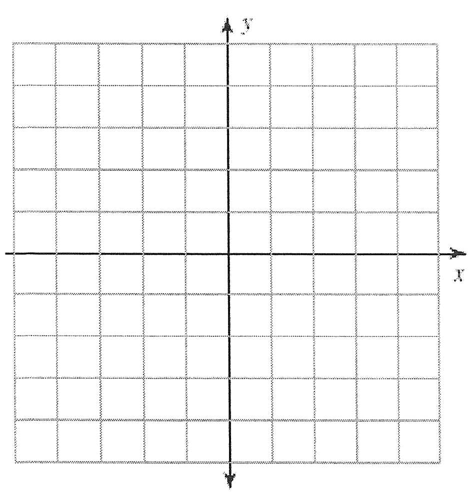


reflection across the x-axis



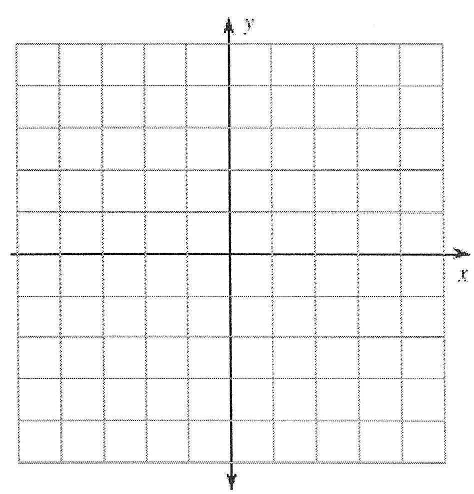
reflection across the x-axis

$T(2, 2)$, $C(2, 5)$, $Z(5, 4)$, $F(5, 0)$



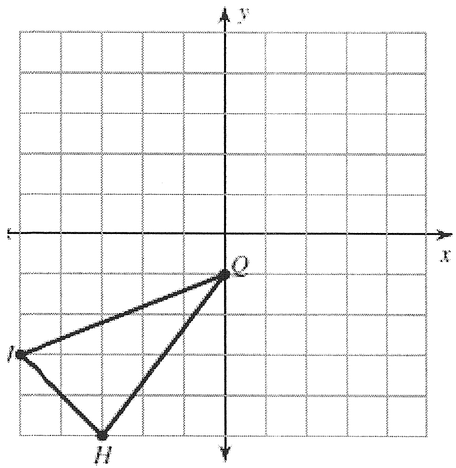
reflection across $y = -2$

$H(-1, -5)$, $M(-1, -4)$, $B(1, -2)$, $C(3, -3)$

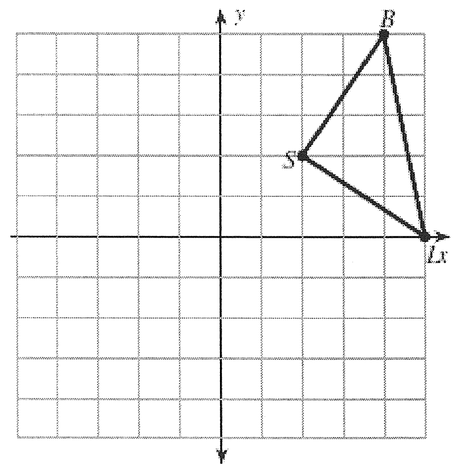


More Practice with Rotations

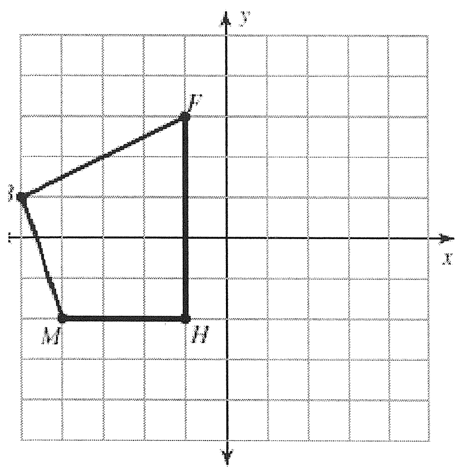
rotation 180° about the origin



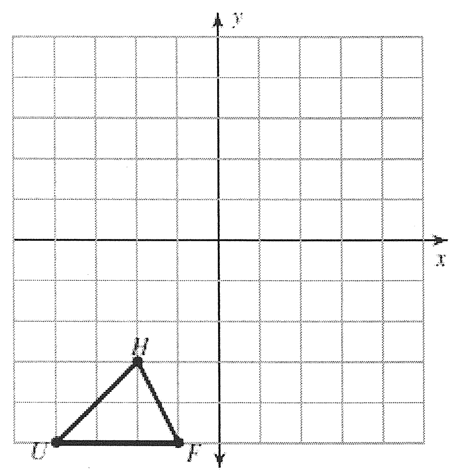
rotation 90° counterclockwise about the origin



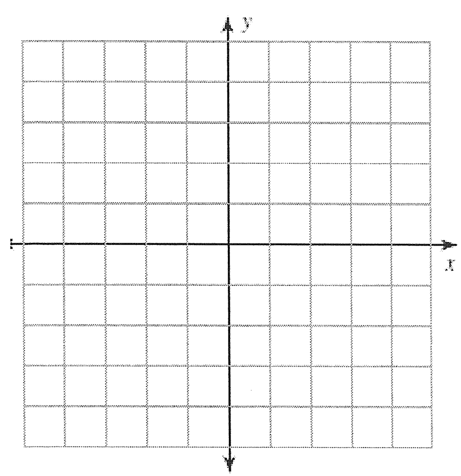
rotation 90° clockwise about the origin



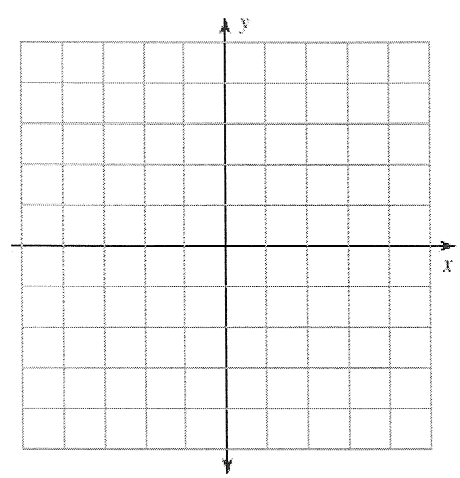
rotation 180° about the origin



rotation 90° clockwise about the origin
$U(1, -2)$, $W(0, 2)$, $K(3, 2)$, $G(3, -3)$



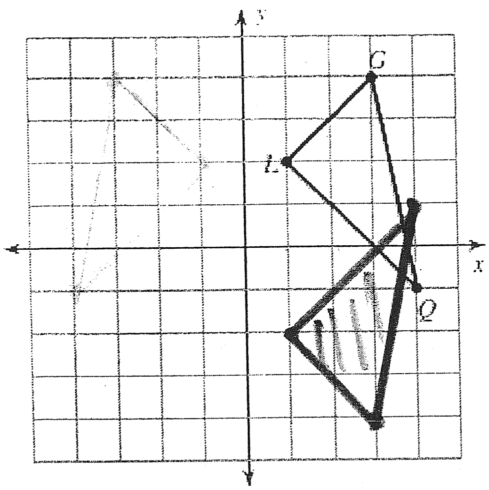
rotation 180° about the origin
$V(2, 0)$, $S(1, 3)$, $G(5, 0)$



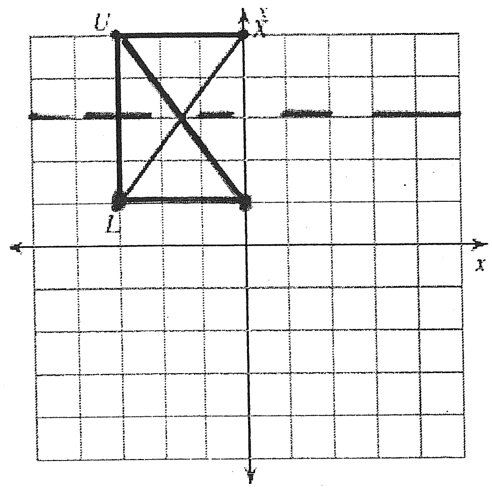
Name: Key Date: ________________ Period: _____

Practice with Reflections & Rotations

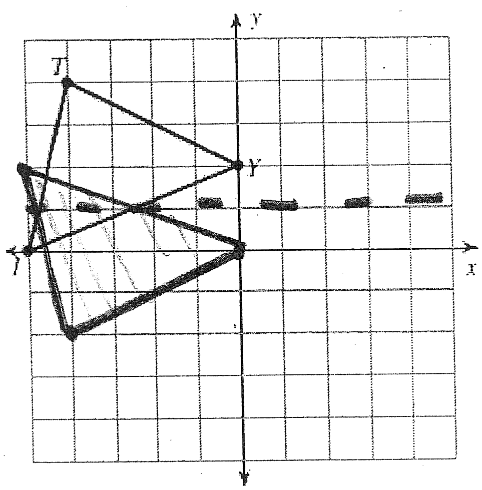
reflection across the x-axis



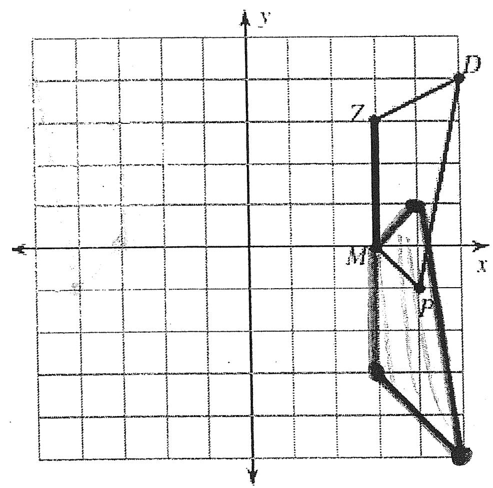
reflection across $y = 3$



reflection across $y = 1$

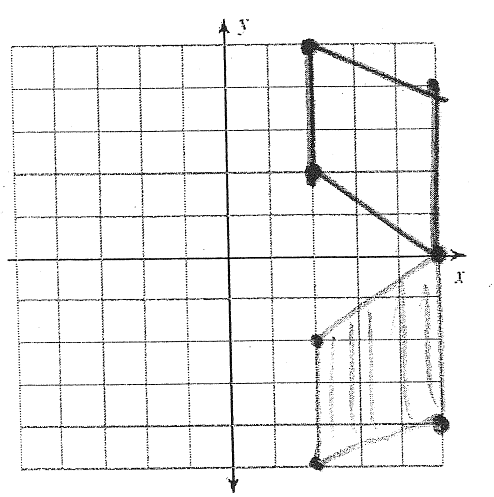


reflection across the x-axis



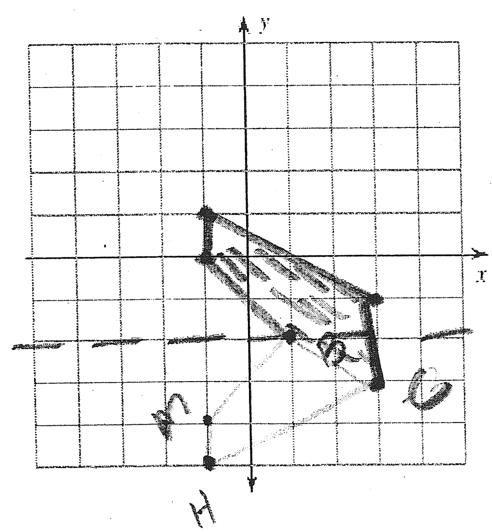
reflection across the x-axis

$T(2, 2)$, $C(2, 5)$, $Z(5, 4)$, $F(5, 0)$



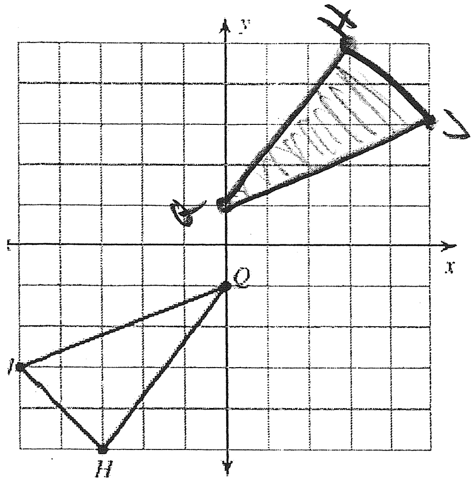
reflection across $y = -2$

$H(-1, -5)$, $M(-1, -4)$, $B(1, -2)$, $C(3, -3)$

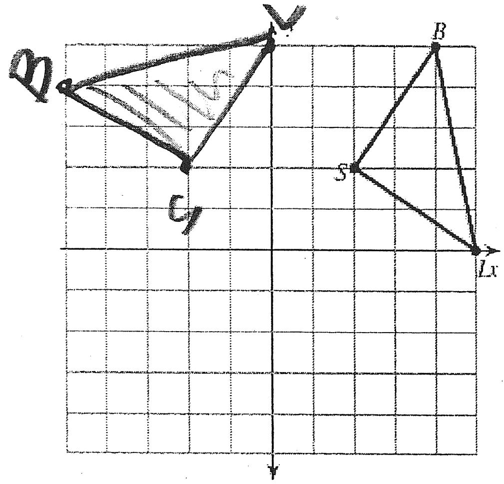


More Practice with Rotations

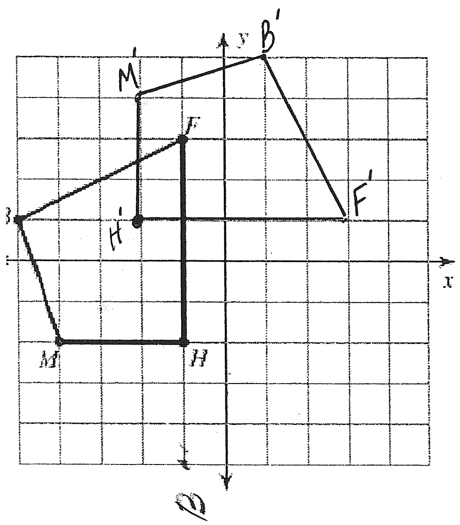
rotation 180° about the origin



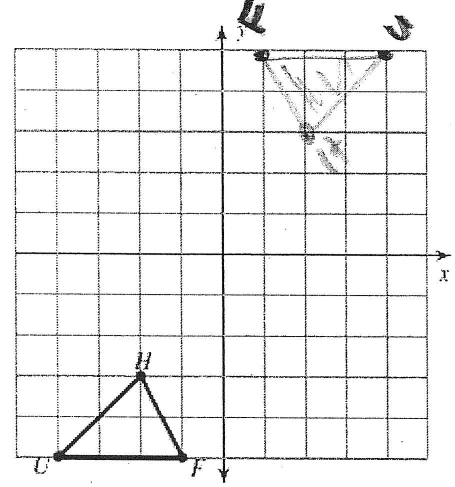
rotation 90° counterclockwise about the origin



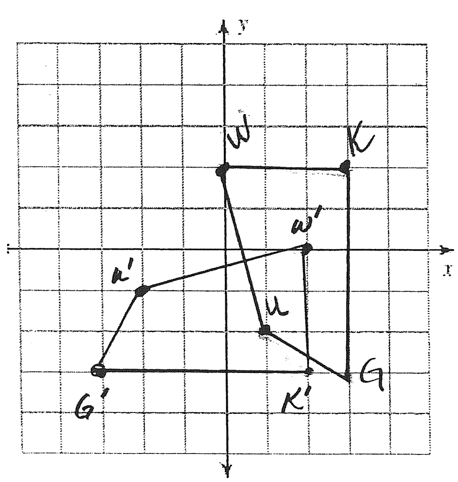
rotation 90° clockwise about the origin



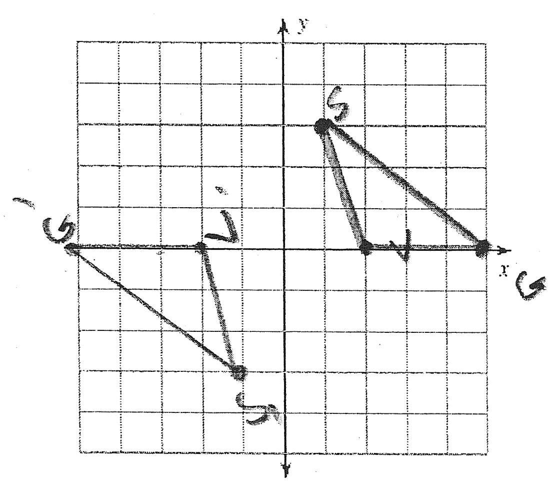
rotation 180° about the origin



rotation 90° clockwise about the origin
$U(1, -2)$, $W(0, 2)$, $K(3, 2)$, $G(3, -3)$



rotation 180° about the origin
$V(2, 0)$, $S(1, 3)$, $G(5, 0)$



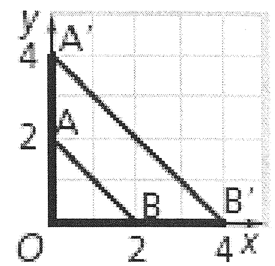
Name ______________________________ Date ________________________ Period _____

KHM Additional Lesson – Dilations

A **dilation** is a transformation of a figure that changes its size but not its shape. The **scale factor** of a dilation determines the extent of the change in size. A dilation is an enlargement when the scale factor is greater than 1. It is a reduction a the scale factor is less than 1.When you dilate a figure, you are either shrinking or enlarging an original figure toward or farther from another point called the **center of dilation.**

The picture below shows the dilation of *AOB* to *A′OB′* with the center of dilation at the origin. The naming of a point like *A′* (ay–prime) signals that *A′* is the new position of *A* after the transformation.

A. Is *A′OB′* an enlargement or a reduction of *AOB*? ____________



B. **1.** How many times greater is *OA′* than *OA*? ________

2. How many times greater is *OB′* than *OB*? ________

3. How many times greater is *A′B′* than *AB*? ________

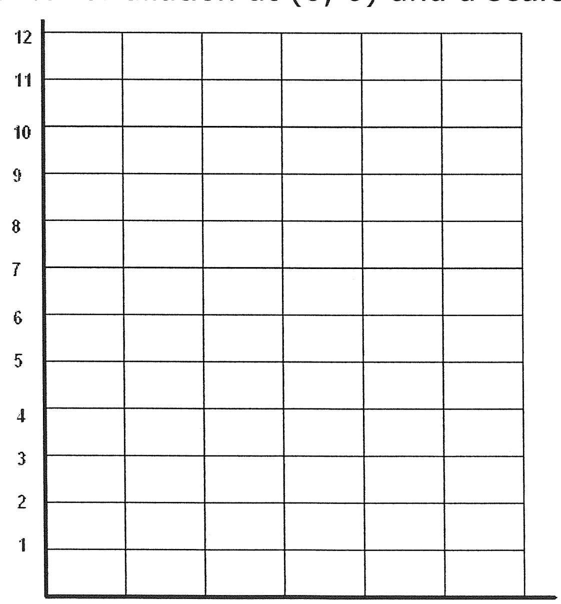
4. The scale factor in a dilation measures the comparative size of linear measures in a figure before and after dilation. What is the scale factor of this dilation?

5. When you are examining a dilation, what is the least information you need in order to determine the scale factor?

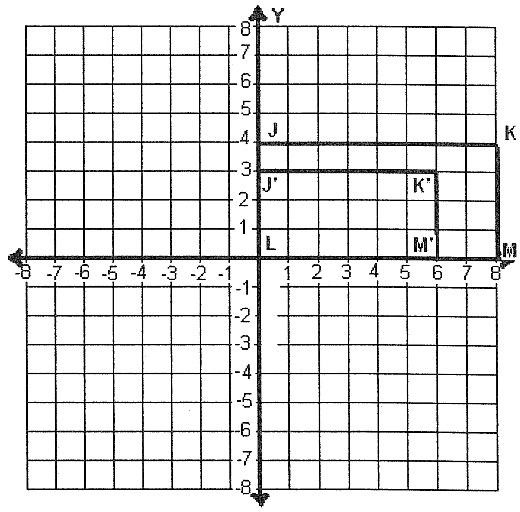
C. How did the center of dilation change position in the dilation of *AOB*?

D. Draw *LOM* with vertices *L*(0, 4),*O*(0, 0), and *M*(2, 0).

Now draw *L′OM′* as a dilation of *LOM* with the center of dilation at (0, 0) and a scale factor of 1.5.



The graph shows the dilation of figure JKLM to J'K'L'M' with the center of dilation at $L(0, 0)$ and a scale factor of ¾.



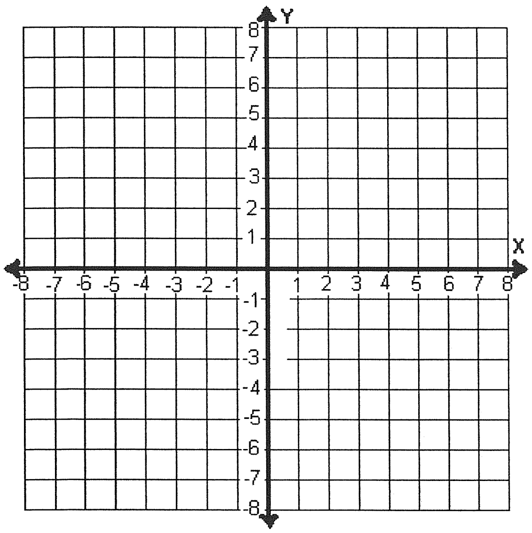
A. Is J'K'L'M' an enlargement or a reduction of JKLM?

B. **1.** What is the ratio of side K'M' to side KM? __________

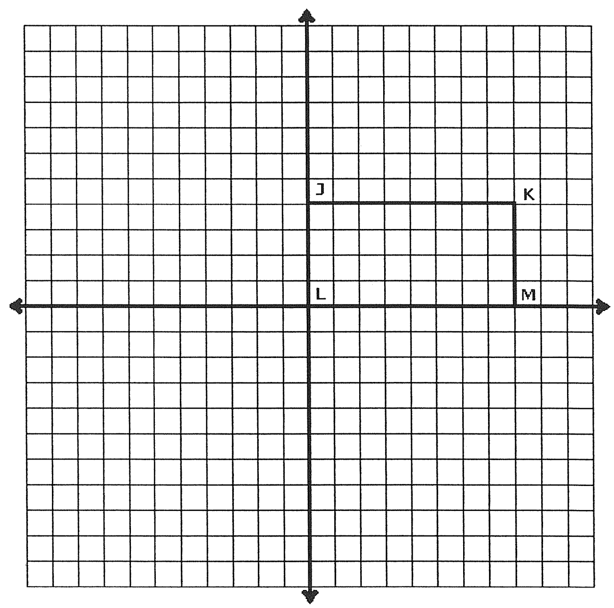
2. What is the ratio of the length of line J'K' to the length of line JK? ___________

C. **1.** Make a copy of JKLM on the graph below.

2. Now draw a reduction of JKLM with the center of dilation at (0, 0) and a scale factor of ¼.

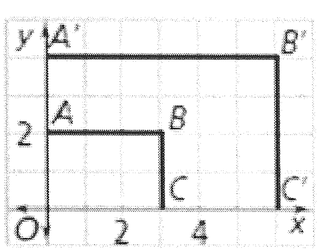


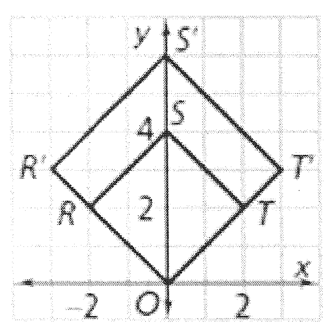
3. Below is a copy of JKLM. Now draw an enlargement of JKLM with a center of dilation at (0,0) and a scale factor of 1.25.



Let's do some more practice!

1. Identify each as an enlargement or reduction. Name the location of the center of dilation and give the scale factor.





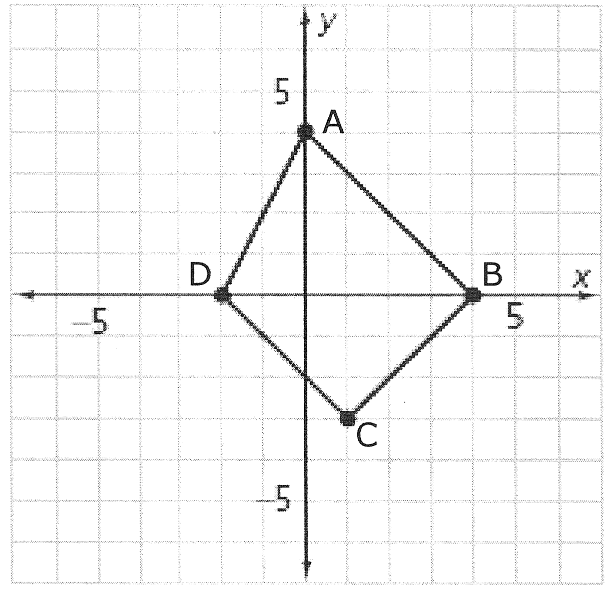
Enlargement or Reduction?

Scale Factor: ___________

Enlargement or Reduction?

Scale Factor: ___________

2. For a dilation centered at the origin you can find the location of points on the dilated image by multiplying the coordinates on the original image by the scale factor.



What are the coordinate for each point?

Point A ___________ Point B __________

Point C ___________ Point D __________

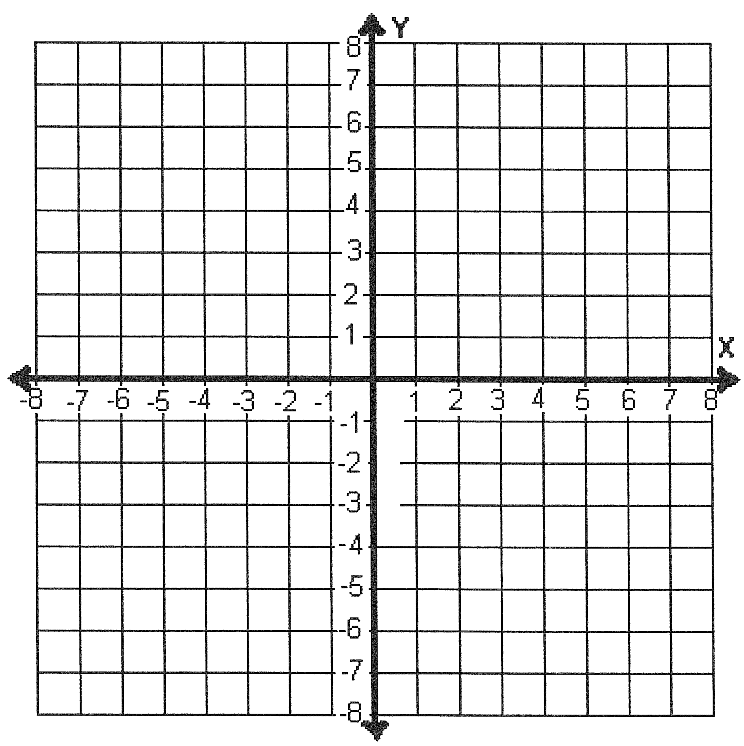
Use a scale factor of $\frac{3}{2}$ to find the new coordinates of our dilated image. Write the new coordinates below.

Point A' ___________ Point B' __________

Point C' ___________ Point D' __________

3. **a.** Draw ΔABC with vertices at (-5,-1), (1, 3), and (1,-1).

b. Dilate ΔABC with a scale factor of ½ and a center of dilation at (0, 0).



Name Keng Date Period

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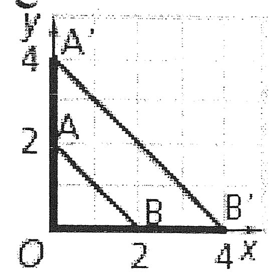
The picture below shows the dilation of *AOB* to *A'OB'* with the center of dilation at the origin. The naming of a point like *A'* (ay–prime) signals that *A'* is the new position of *A* after the transformation.

A. Is *A'OB'* an enlargement or a reduction of *AOB*? enlargement



B. **1.** How many times greater is *OA'* than *OA*? 2

2. How many times greater is *OB'* than *OB*? 2

3. How many times greater is *A'B'* than *AB*? 2

4. The scale factor in a dilation measures the comparative size of linear measures in a figure before and after dilation. What is the scale factor of this dilation?

2

5. When you are examining a dilation, what is the least information you need in order to determine the scale factor?

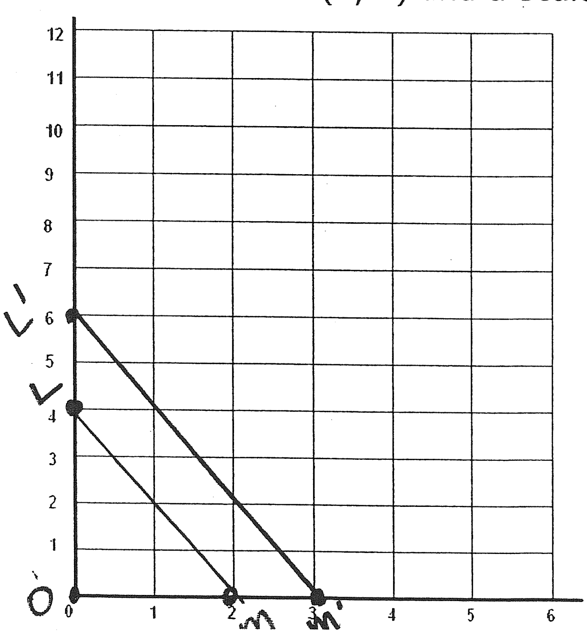
length of 2 corresponding sides

C. How did the center of dilation change position in the dilation of *AOB*?

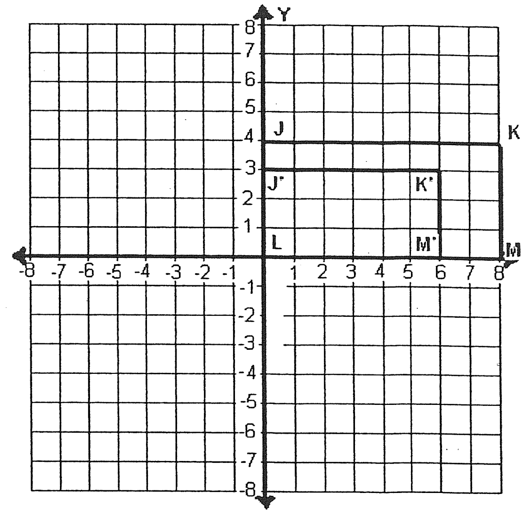
It stayed the same

D. Draw *LOM* with vertices *L*(0, 4),*O*(0, 0), and *M*(2, 0).

Now draw *L'OM'* as a dilation of *LOM* with the center of dilation at (0, 0) and a scale factor of 1.5.



The graph shows the dilation of figure JKLM to J'K'L'M' with the center of dilation at $L(0, 0)$ and a scale factor of ¾.



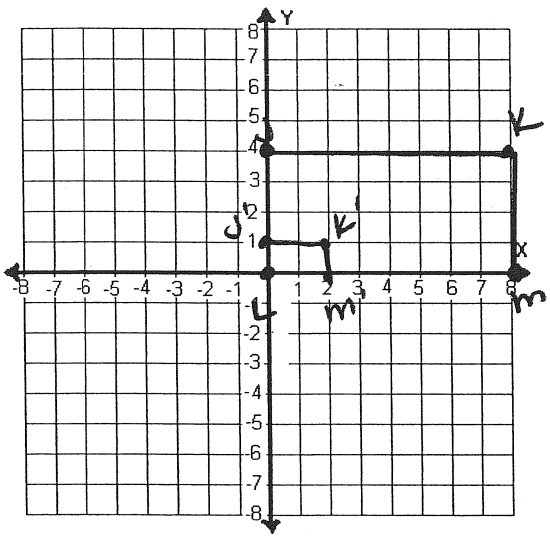
A. Is J'K'L'M' an enlargement or a reduction of JKLM?

B. 1. What is the ratio of side K'M' to side KM? 3:4 or $\frac{3}{4}$

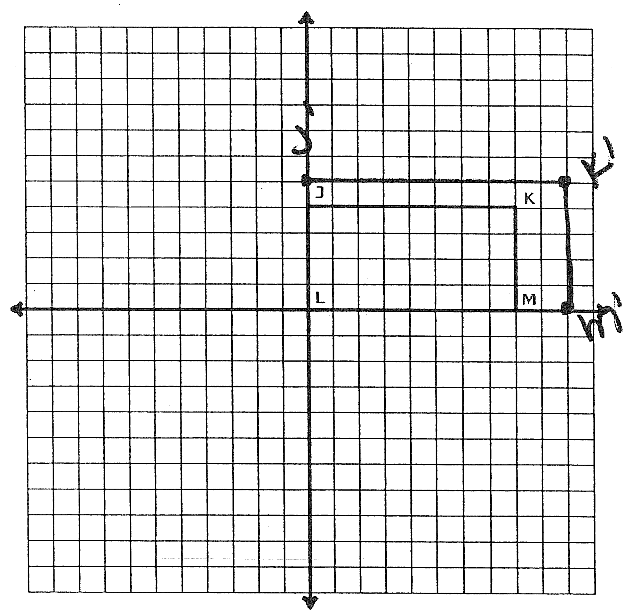
2. What is the ratio of the length of line J'K' to the length of line JK? 3:4 or $\frac{3}{4}$

C. 1. Make a copy of JKLM on the graph below.

2. Now draw a reduction of JKLM with the center of dilation at (0, 0) and a scale factor of ¼.

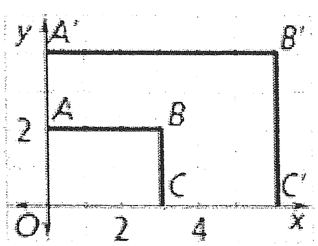


3. Below is a copy of JKLM. Now draw an enlargement of JKLM with a center of dilation at (0,0) and a scale factor of 1.25.



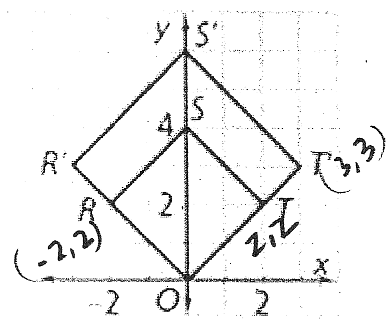
Let's do some more practice!

1. Identify each as an enlargement or reduction. Name the location of the center of dilation and give the scale factor.



Enlargement or Reduction?

Scale Factor: 2

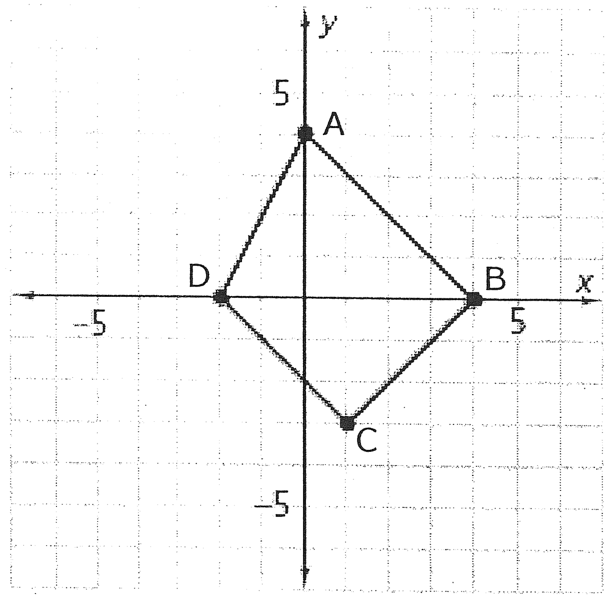


(0,4)
0,6)

Enlargement or Reduction?

Scale Factor: 1.5

2. For a dilation centered at the origin you can find the location of points on the dilated image by multiplying the coordinates on the original image by the scale factor.



What are the coordinate for each point?

Point A (0,4) Point B (4,0)

Point C (1,-3) Point D (0,-2)

Use a scale factor of $\frac{3}{2}$ to find the new coordinates of our dilated image. Write the new coordinates below.

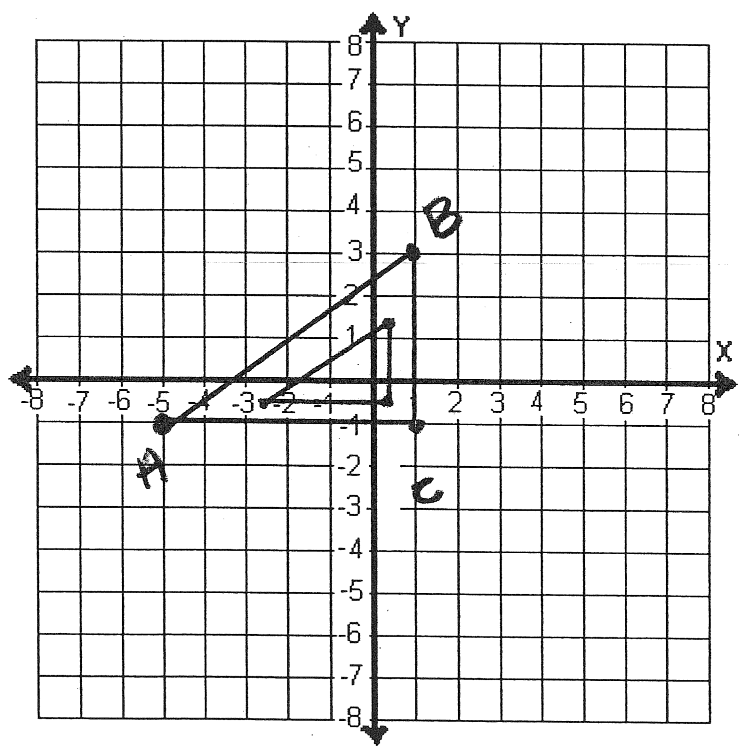
Point A' (0,6) Point B' (6,0)

Point C' (1.5,-4.5) Point D' (0,-3)

3. **a.** Draw ΔABC with vertices at (-5,-1), (1, 3), and (1,-1). (-5,-1) × $\frac{1}{2}$ (-2.5, -$\frac{1}{2}$)

(½,1½) (½,-½)

b. Dilate ΔABC with a scale factor of ½ and a center of dilation at (0, 0).



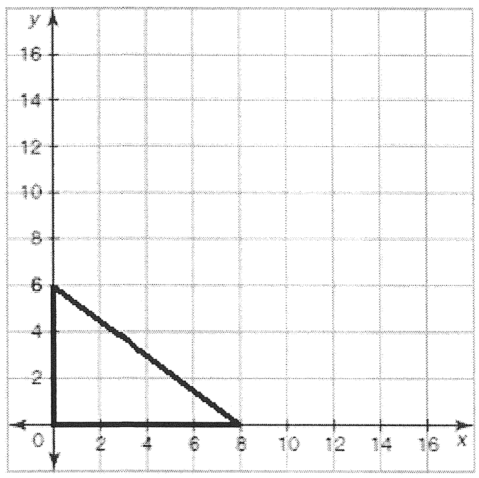
Name ______________________________ **Date** ________________ **Period** ____

Practice – Dilations

Dilate the figures below with the given scale factor and a center of dilation at the origin, (0,0)

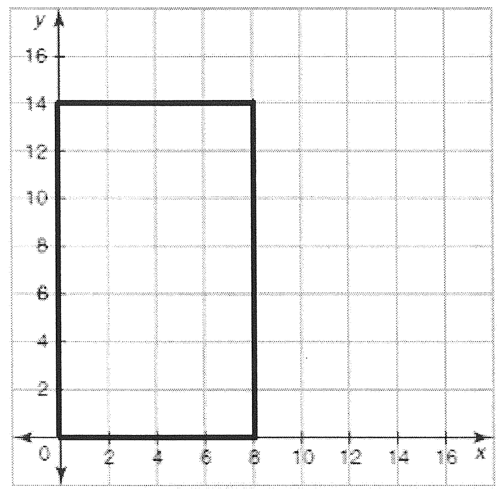
1. Scale Factor of 2 and

   a center of dilation at (0,0)



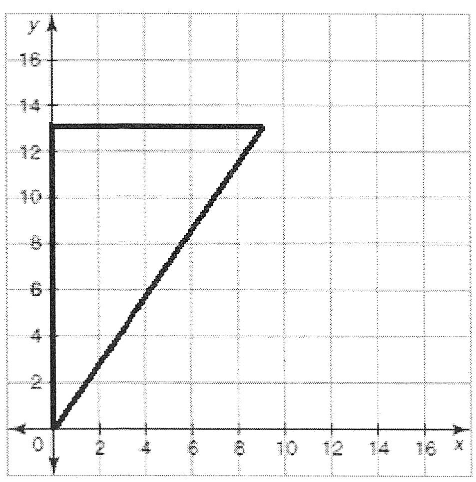
2. Scale Factor of $\frac{1}{2}$ and

   a center of dilation at (0,0)



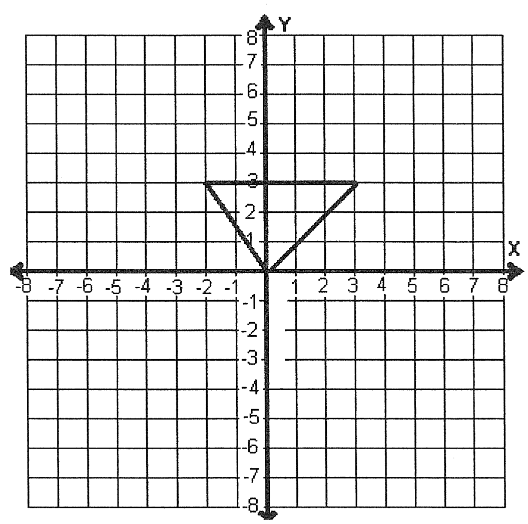
3. Scale Factor of $\frac{2}{3}$ and

   center of dilation at (0,0)



4. Scale Factor of $\frac{3}{2}$ and

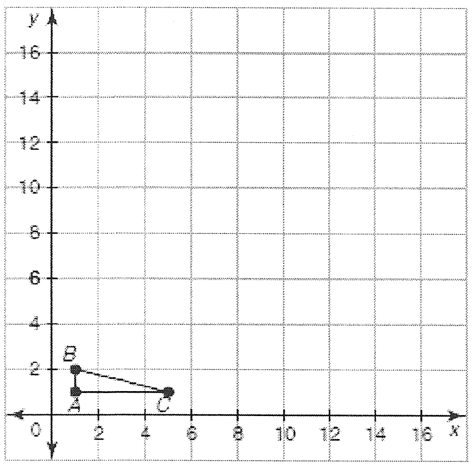
   a center of dilation at (0,0)



5. ΔABC has vertices $A(1, 2)$, $B(3, 6)$, and $C(9, 7)$. What are the vertices of the image after a dilation with a scale factor of 4 using the origin as the center of dilation?

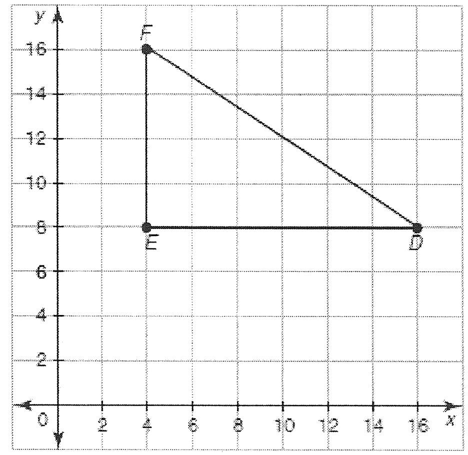
6. Scale Factor of 3 and

center of dilation at (0,0)

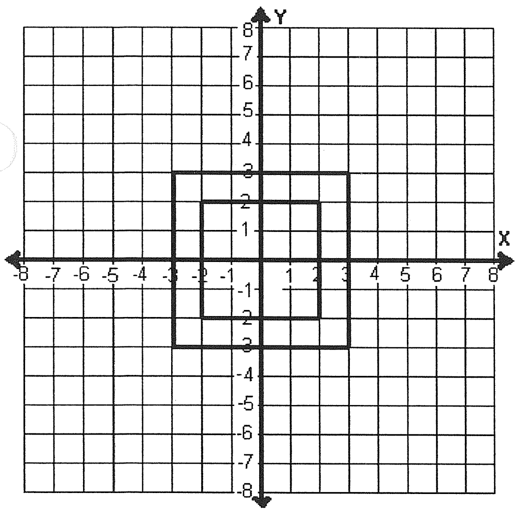


7. Scale Factor of $\frac{1}{4}$ and

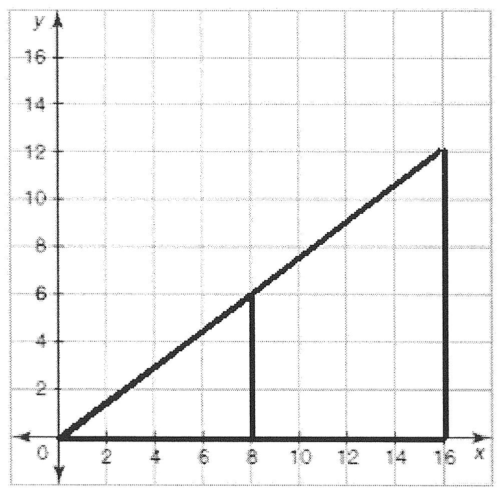
a center of dilation at (0,0)



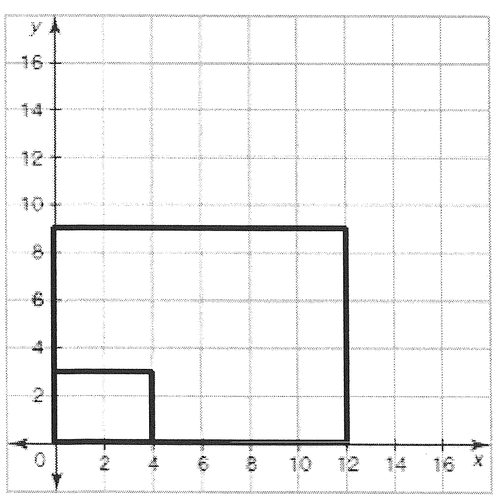
8. Find the scale factor from the large square to the small square.



9. Find the scale factor from the small triangle to the large triangle.



10. Find the scale factor from the large rectangle to the small rectangle.



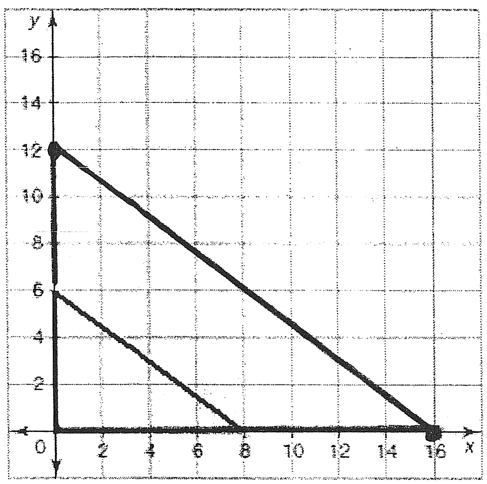
11. ΔABC has vertices $A(1, 2)$, $B(3, 6)$, and $C(9, 7)$. What are the vertices of the image after a dilation with a scale factor of 4 using the origin as the center of dilation?

Name Yeng Date ______________ Period ___

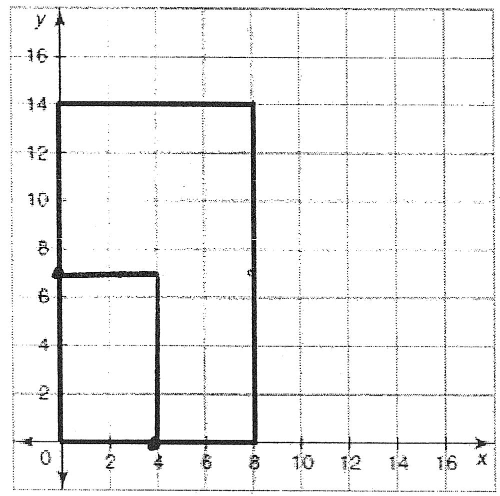
Practice – Dilations

Dilate the figures below with the given scale factor and a center of dilation at the origin, (0,0)

1. Scale Factor of 2 and
 a center of dilation at (0,0)

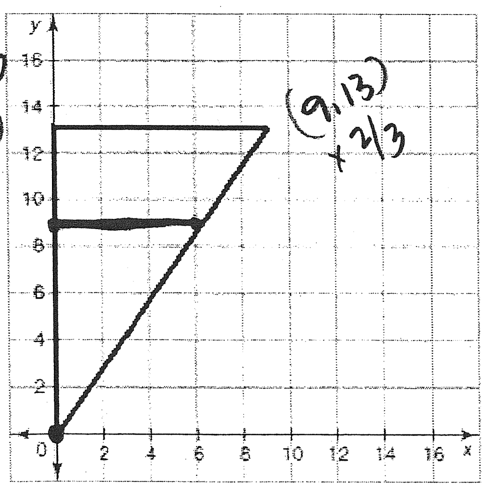


2. Scale Factor of $\frac{1}{2}$ and
 a center of dilation at (0,0)



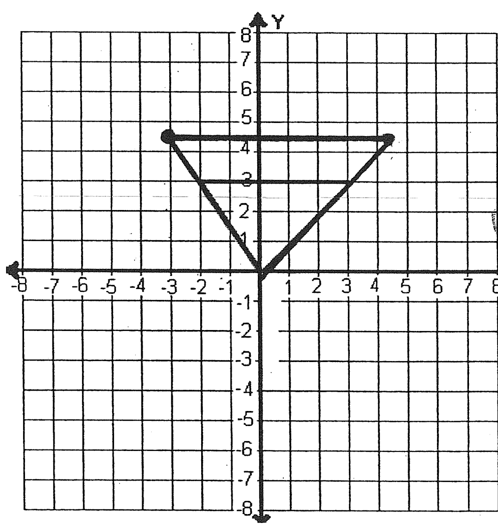
3. Scale Factor of $\frac{2}{3}$ and
 center of dilation at (0,0)

(0,13) x2/3



(9,13) x2/3

4. Scale Factor of $\frac{3}{2}$ and
 a center of dilation at (0,0)



(-2,3) (-3,4½)

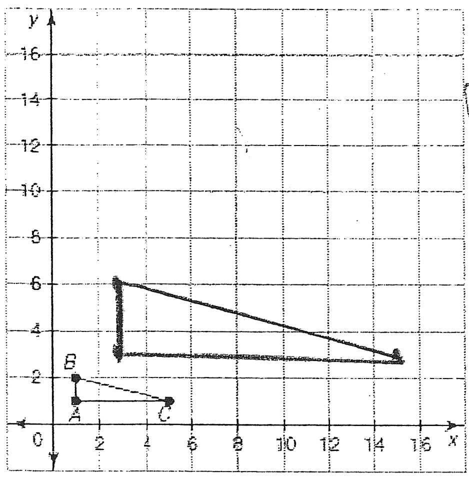
(3,3) (4½,4½)

5. ΔABC has vertices $A(1, 2)$, $B(3, 6)$, and $C(9, 7)$. What are the vertices of the image after a dilation with a scale factor of 4 using the origin as the center of dilation?

A' (4,8) B' (12,24) C' (36,28)

6. Scale Factor of 3 and

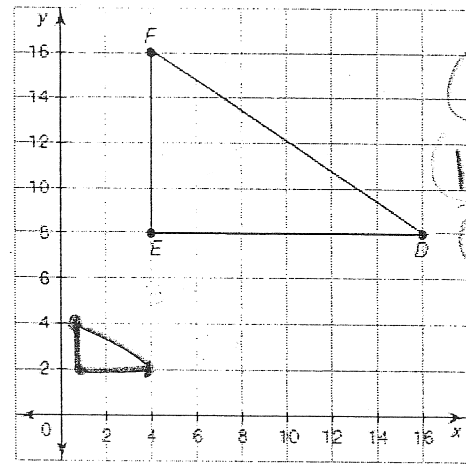
center of dilation at (0,0)



$(1,1) \rightarrow (3,3)$
$(5,1) \rightarrow (15,3)$
$(1,2) \rightarrow (3,6)$

7. Scale Factor of $\frac{1}{4}$ and

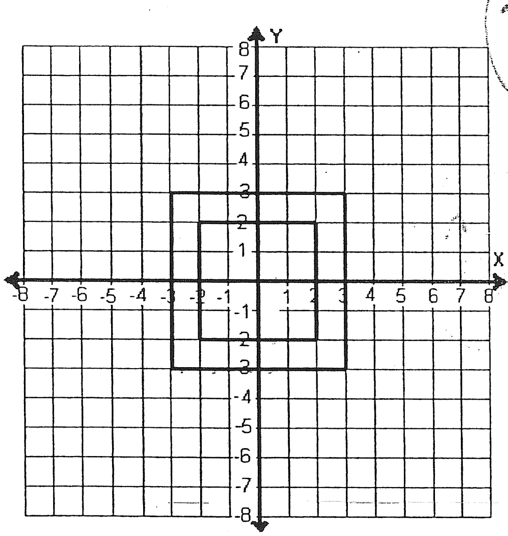
a center of dilation at (0,0)



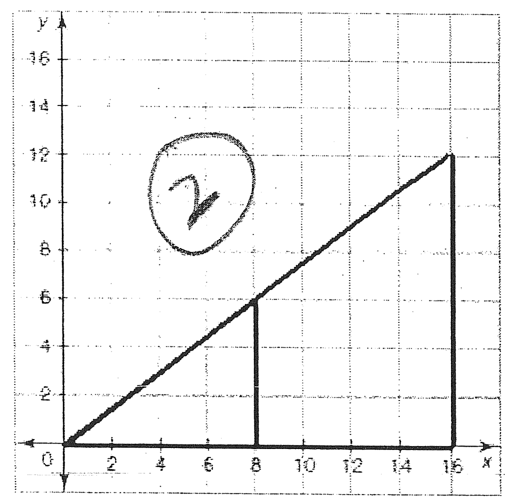
$(4,8) \rightarrow (1,2)$
$(16,8) \rightarrow (4,2)$
$(4,16) \rightarrow (1,4)$

8. Find the scale factor from the large square to the small square.

$2/3$

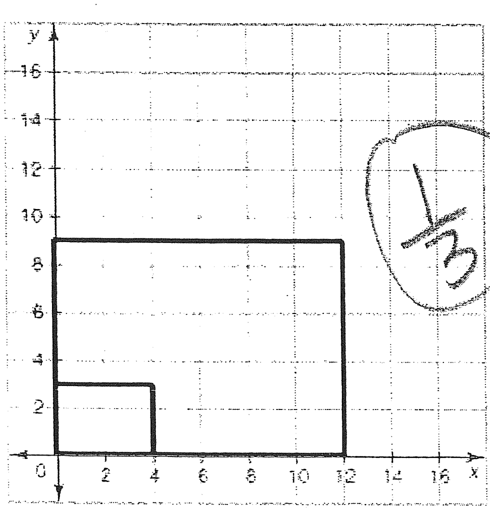


9. Find the scale factor from the small triangle to the large triangle.



2

10. Find the scale factor from the large rectangle to the small rectangle.



$\frac{1}{3}$

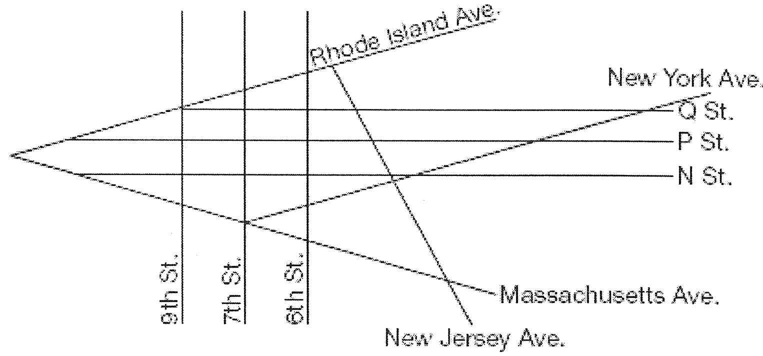
11. ΔABC has vertices $A(1, 2)$, $B(3, 6)$, and $C(9, 7)$. What are the vertices of the image after a dilation with a scale factor of 4 using the origin as the center of dilation?

$A'(4,8)$
$B'(12,24)$
$C'(36,28)$

NAME: ______________________________ **DATE:** ____________ **PERIOD:** _____

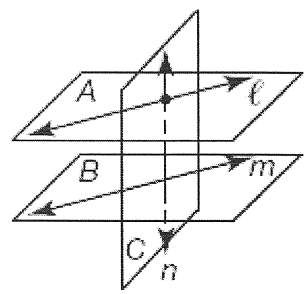
Additional Lesson: Angle Vocabulary w/ Parallel Lines

The layout of the streets of Washington, D.C., was created by Pierre Charles L'Enfant, a French-born architect. L'Enfant began working on the layout of the city in 1791. A map of part of Washington, D.C., is shown to the right.



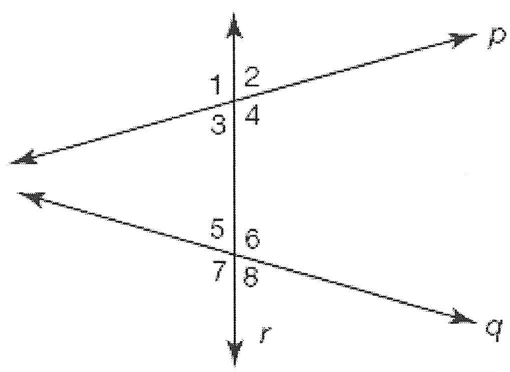
1. **Parallel lines** are lines that do not intersect. Identify three pairs of lines formed by the streets above that appear to be parallel.

2. Line n is called a **transversal** of lines m and ℓ. When a line intersects two or more other lines all at different points, the line is a called a transversal. The transversal is said to "cut" the lines.



How many angles are created when two lines are cut by a transversal? ___________

3. Now consider the lines to the right, formed by Rhode Island Avenue (line p), Massachusetts Avenue (line q), and 9th Street (transversal r).

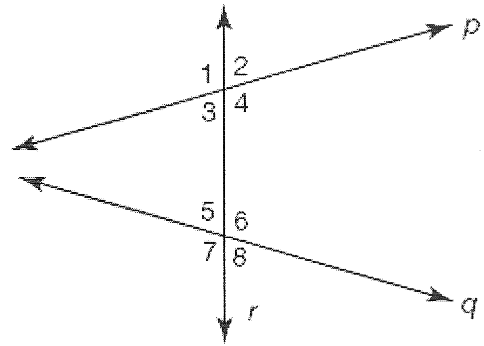


a. We can classify the angles created when two line are cut by a transversal. Four of the eight angles are ***interior angles,*** and the other four angles are ***exterior angles.*** Which four angles do you think are the interior angles? Explain how you know.

b. Which four angles do you think are exterior angles? Explain how you know.

Let's look at some other ways to classify these angles.

4. Two angles are **alternate interior angles** if they lie between the two lines on opposite (alternate) sides of the transversal. $\angle 3$ and $\angle 6$ in the figure at the right are alternate interior angles.

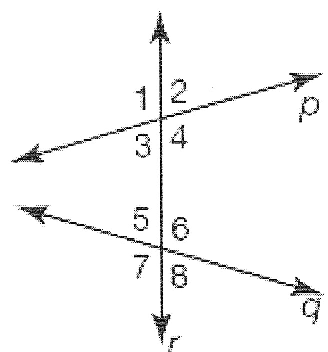


Name any other pairs of alternate interior angles given in the figure.

5. Two angles are **alternate exterior angles** if they lie outside the two lines on opposite (alternate) sides of the transversal. $\angle 1$ and $\angle 8$ in the figure above are alternate exterior angles.

Name any other pairs of alternate exterior angles given in the figure.

6. Two angles are **corresponding angles** if they are on the same side of the transversal in corresponding positions. $\angle 1$ and $\angle 5$ in the figure at the right are corresponding angles.



Name any other pairs of corresponding angles given in the figure.

7. Use the figure above at the right to answer the following questions.

Suppose that $m\angle 5 = 70^{\circ}$ in the figure above at the left.

What is $m\angle 6$? ________ $m\angle 7$? ___________ $m\angle 8$? ___________

a. What do you notice about the measures of $\angle 5$ and $\angle 6$?

b. What do you notice about the measures of $\angle 5$ and $\angle 7$?

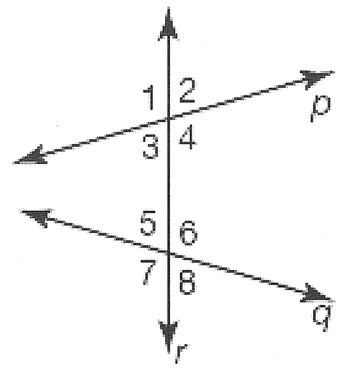
These pairs of angles are called **supplementary angles**. Supplementary angles always have a sum of 180°.

c. What do you notice about the measures of $\angle 5$ and $\angle 8$?

Angle 5 and $\angle 8$ are examples of **vertical angles**. Vertical angles always have the same angle measures.

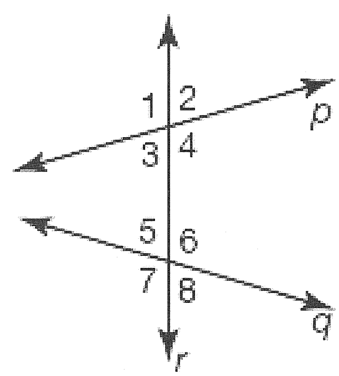
8. Suppose the $m\angle 1 = 115^{\circ}$ in the figure to the right. Use this information to find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

$m\angle 2 =$ ________ $m\angle 3 =$ ________ $m\angle 4 =$ ________



9. List all the pairs of supplementary angles formed by lines *p* and *q* and transversal *r*.

What is the relationship between the measures of the angles in each pair?



10. List all the pairs of vertical angles formed by the lines and the transversal.

What is the relationship between the measures of the angles in each pair?

11. Look at the two diagrams below.

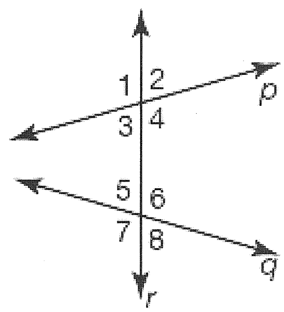


Diagram1

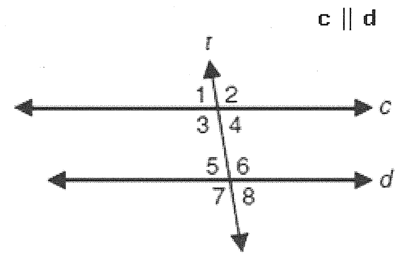


Diagram 2

a) Name a pair of corresponding angles from Diagram 1.

c) What do you notice about the measures of these two angles?

e) Names a pair of alternate interior angles from Diagram 1.

g) What do you notice about the measures of these two angles?

i) Names a pair of alternate exterior angles from Diagram 1.

k) What do you notice about the measures of these two angles?

b) Name a pair of corresponding angles from Diagram 2.

d) What do you notice about the measures of these two angles?

f) Names of pair of alternate interior angles from Diagram 2.

h) What do you notice about the measures of these two angles?

j) Names of pair of alternate exterior angles from Diagram 2.

l) What do you notice about the measures of these two angles?

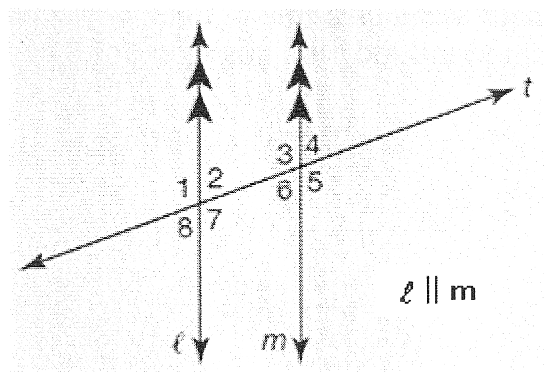
m) Look at your responses for parts (c), (g), (k) and parts (d), (h), (l). What is different about the responses for Diagram 1 versus the responses for Diagram 2?

n) What is the difference between Diagram 1 and Diagram 2 that would make this happen?

When parallel lines are cut by a transversal, pairs of
______________________ angles, pairs of ______________________
__________________ angles, and pairs of ______________________
__________________ angles are congruent.

12. Consider the lines below, formed from Rhode Island Avenue (transversal t), 9th Street (line ℓ), and 7th Street (line m). Lines ℓ and m are parallel. This is indicated on this figure by using double arrows. (➤➤).

Find the missing angle measures. Be ready to explain how you got your answers.



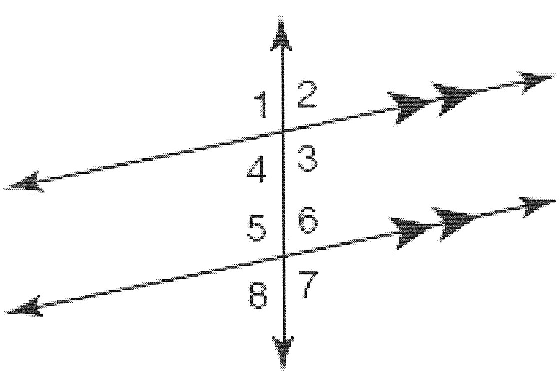
$m\angle 1 = 110^{o}$ $m\angle 2 =$ _______

$m\angle 3 =$ _______ $m\angle 4 =$ _______

$m\angle 5 =$ _______ $m\angle 6 =$ _______

$m\angle 7 =$ _______ $m\angle 8 =$ _______

13. Two lines shown in the figure are parallel cut by a transversal. The measure of $\angle 1$ is 95^{o}. Find the missing angle measures without using a protractor. Be ready to explain how you got your answers.



$m\angle 2 =$ ________ $m\angle 3 =$ ________

$m\angle 4 =$ ________ $m\angle 5 =$ ________

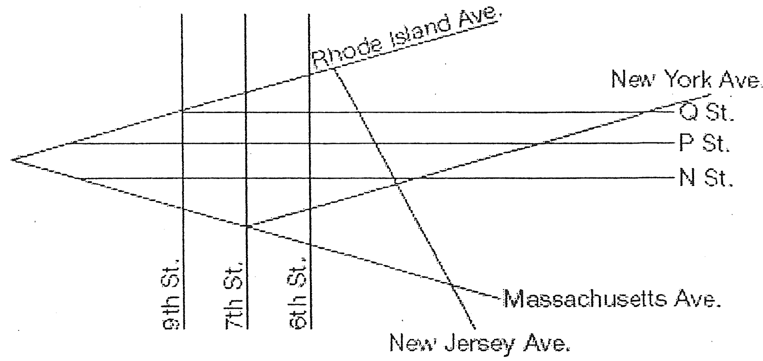
$m\angle 6 =$ ________ $m\angle 7 =$ ________

$m\angle 8 =$ ________

NAME: ______________________________ **DATE:** ____________ **PERIOD:** _____

Additional Lesson: Angle Vocabulary w/ Parallel Lines

The layout of the streets of Washington, D.C., was created by Pierre Charles L'Enfant, a French-born architect. L'Enfant began working on the layout of the city in 1791. A map of part of Washington, D.C., is shown to the right.

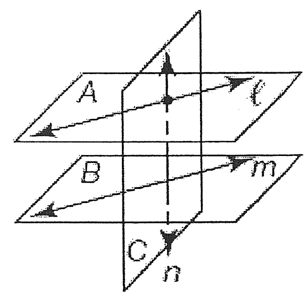


1. **Parallel lines** are lines that do not intersect. Identify three pairs of lines formed by the streets above that appear to be parallel.

S & P   P & N   Q & N

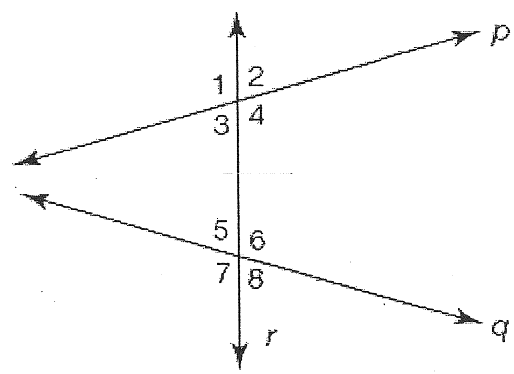
9th & 6th   9th & 7th   7th & 6th

2. Line *n* is called a **transversal** of lines *m* and *ℓ*. When a line intersects two or more other lines all at different points, the line is a called a transversal. The transversal is said to "cut" the lines.



How many angles are created when two lines are cut by a transversal? ___8___

3. Now consider the lines to the right, formed by Rhode Island Avenue (line *p*), Massachusetts Avenue (line *q*), and 9th Street (transversal *r*).



a. We can classify the angles created when two line are cut by a transversal. Four of the eight angles are ***interior angles***, and the other four angles are ***exterior angles.*** Which four angles do you think are the interior angles? Explain how you know.

∠3, ∠4, ∠5, ∠6

They are inside the two lines

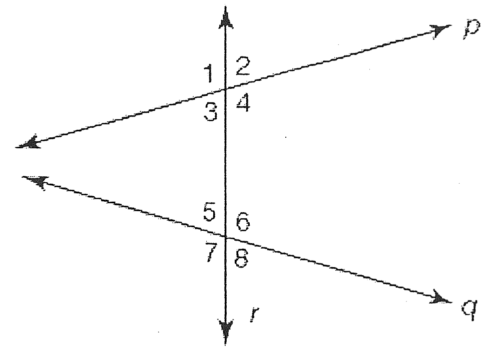
b. Which four angles do you think are exterior angles? Explain how you know.

∠1, ∠2, ∠7, ∠8

They are outside the two lines

Let's look at some other ways to classify these angles.

4. Two angles are **alternate interior angles** if they lie between the two lines on opposite (alternate) sides of the transversal. $\angle 3$ and $\angle 6$ in the figure at the right are alternate interior angles.



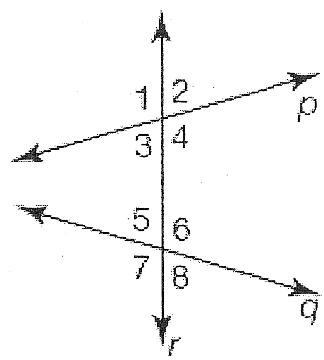
Name any other pairs of alternate interior angles given in the figure. $\angle 4$ & $\angle 5$

5. Two angles are **alternate exterior angles** if they lie outside the two lines on opposite (alternate) sides of the transversal. $\angle 1$ and $\angle 8$ in the figure above are alternate exterior angles.

Name any other pairs of alternate exterior angles given in the figure.

$\angle 2$ & $\angle 7$

6. Two angles are **corresponding angles** if they are on the same side of the transversal in corresponding positions. $\angle 1$ and $\angle 5$ in the figure at the right are corresponding angles.



Name any other pairs of corresponding angles given in the figure.

$\angle 3$ & $\angle 7$, $\angle 2$ & $\angle 6$,
$\angle 4$ & $\angle 8$

7. Use the figure above at the right to answer the following questions.

Suppose that $m\angle 5 = 70^{\circ}$ in the figure above at the left.

What is $m\angle 6$? $\underline{110^{\circ}}$ $m\angle 7$? $\underline{110^{\circ}}$ $m\angle 8$? $\underline{70^{\circ}}$

a. What do you notice about the measures of $\angle 5$ and $\angle 6$?

add up to 180°

b. What do you notice about the measures of $\angle 5$ and $\angle 7$?

add up to 180°

These pairs of angles are called **supplementary angles**. Supplementary angles always have a sum of 180°.

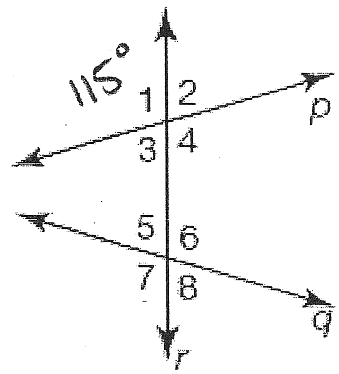
c. What do you notice about the measures of $\angle 5$ and $\angle 8$?

same measure

Angle 5 and $\angle 8$ are examples of **vertical angles**. Vertical angles always have the same angle measures.

8. Suppose the $m\angle 1 = 115^{o}$ in the figure to the right. Use this information to find $m\angle 2$, $m\angle 3$, and $m\angle 4$.

$m\angle 2 =$ $\underline{65^\circ}$ $m\angle 3 =$ $\underline{65^\circ}$ $m\angle 4 =$ $\underline{115^\circ}$

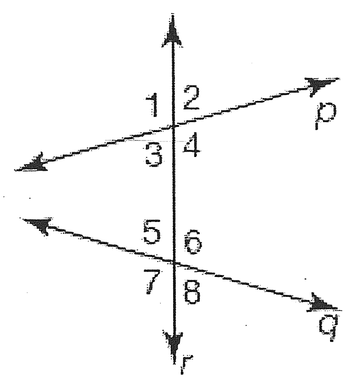


9. List <u>all</u> the pairs of supplementary angles formed by lines *p* and *q* and transversal *r*.

∠1 & ∠2 ∠2 & ∠4
∠1 & ∠3 ∠3 & ∠4
∠5 & ∠6 ∠6 & ∠8
∠7 & ∠8 ∠5 & ∠7

What is the relationship between the measures of the angles in each pair?

They add up to 180°



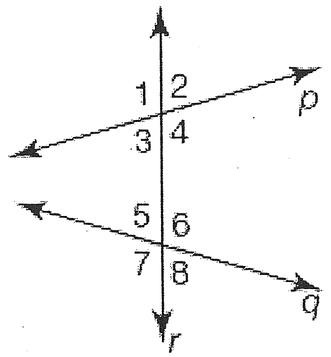
10. List <u>all</u> the pairs of vertical angles formed by the lines and the transversal.

∠1 & ∠4 ∠2 & ∠3
∠5 & ∠8 ∠6 & ∠7

What is the relationship between the measures of the angles in each pair?

They have the same measure.

11. Look at the two diagrams below.



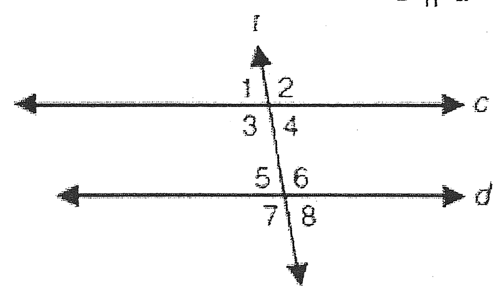


Diagram 1	**Diagram 2**
Name a pair of corresponding gles from Diagram 1. ex: ∠1 & ∠5	Name a pair of corresponding gles from Diagram 2. ex. ∠1 & ∠5
What do you notice about the easures of these two angles? not the same measure.	What do you notice about the easures of these two angles? same measure
Name a pair of alternate interior gles from Diagram 1. ex: ∠3 & ∠6	Name a pair of alternate interior gles from Diagram 2. ex. ∠4 & ∠5
What do you notice about the easures of these two angles? not the same measure	What do you notice about the easures of these two angles? same measure
Name a pair of alternate exterior gles from Diagram 1. ∠1 & ∠8	Name a pair of alternate exterior gles from Diagram 2. ex: ∠1 & ∠8
What do you notice about the easures of these two angles? not the same measure	What do you notice about the easures of these two angles? same measure.

) Look at your responses for parts (c), (g), (k) and parts (d), (h), (l). What is different out the responses for Diagram 1 versus the responses for Diagram 2?

diagram 1: angle measures are not the same
diagram 2: angle measures are the same,

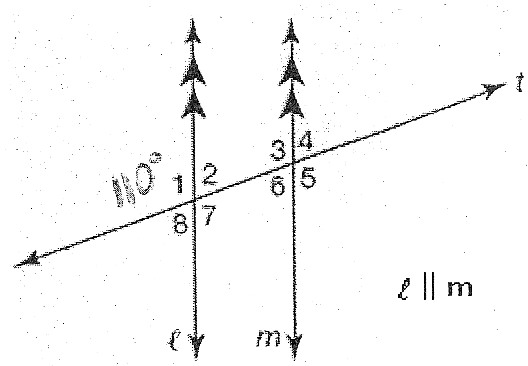
What is the difference between Diagram 1 and Diagram 2 that would make this ppen?

diagram 2 has parallel lines!

When parallel lines are cut by a transversal, pairs of corresponding angles, pairs of alternate interior angles, and pairs of alternate exterior angles are congruent.

12. Consider the lines below, formed from Rhode Island Avenue (transversal *t*), 9th Street (line *ℓ*), and 7th Street (line *m*). Lines *ℓ* and m are parallel. This is indicated on this figure by using double arrows. (➤➤).

Find the missing angle measures. Be ready to explain how you got your answers.



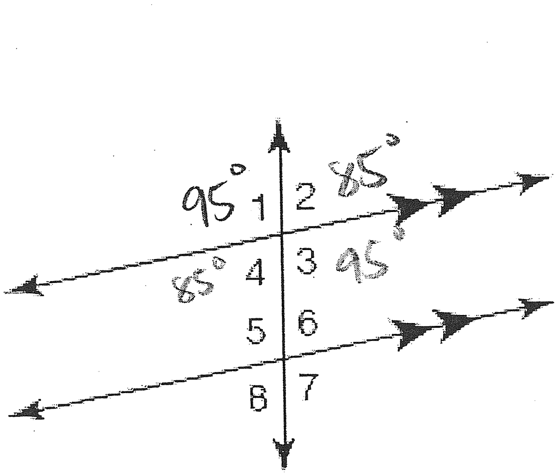
$m\angle 1 = 110^\circ$ $m\angle 2 =$ 70°

$m\angle 3 =$ 110° $m\angle 4 =$ 70°

$m\angle 5 =$ 110° $m\angle 6 =$ 70°

$m\angle 7 =$ 110° $m\angle 8 =$ 70°

13. Two lines shown in the figure are parallel cut by a transversal. The measure of $\angle 1$ is 95°. Find the missing angle measures without using a protractor. Be ready to explain how you got your answers.



$m\angle 2 =$ 85° $m\angle 3 =$ 95°

$m\angle 4 =$ 85° $m\angle 5 =$ 95°

$m\angle 6 =$ 85° $m\angle 7 =$ 95°

$m\angle 8 =$ 85°

Name ____ Date ____________ Class ____________

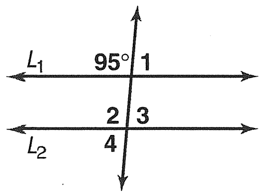
Skill: Angles and Parallel Lines

Investigation 2

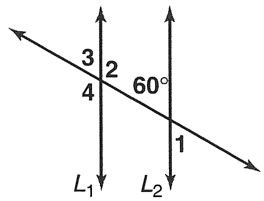
Shapes and Designs

In each diagram below, lines L_1 and L_2 are parallel lines cut by a transversal. Find the measure of each numbered angle.

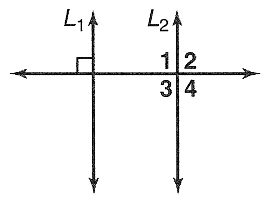
1.



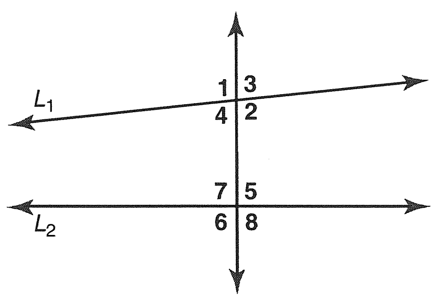
2.



3.



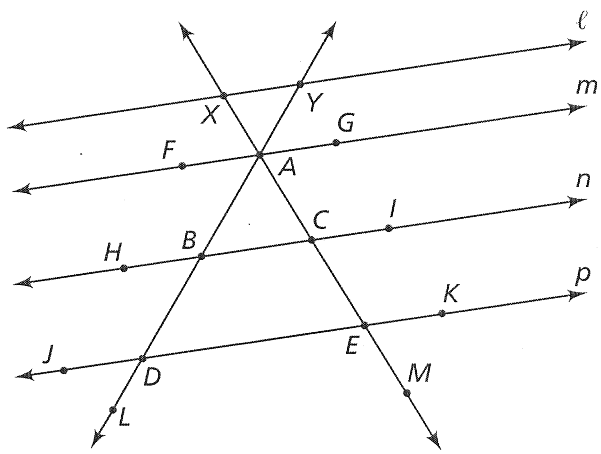
4. Use the figure below. Is line L_1 parallel to line L_2? Explain how you could use an angle ruler to support your conjecture.



Topic 6: Parallel and Perpendicular

for use after ***The Shapes of Algebra*** **Investigation 2**

In the diagram below, lines ℓ, m, n, and p are parallel lines. The other two lines are **transversals.** Angles ACI and CEK are in corresponding positions at the vertices C and E. Each is in the "top-right" or "north-east" position at their vertices. Because of their corresponding positions, they are called **corresponding angles.** Corresponding angles are congruent to each other if they are formed by a transversal intersecting parallel lines. Angles BAC and XAY are **vertical angles.** Vertical angles are always congruent to each other.



When parallel lines are cut by non-parallel transversals, similar triangles are formed. In this figure, triangle ABC is similar to triangle ADE. Corresponding sides of these similar triangles form equal ratios.
For example:

$$\frac{\text{length of } \overline{AC}}{\text{length of } \overline{AE}} = \frac{\text{length of } \overline{AB}}{\text{length of } \overline{AD}}$$

Exercises

For Exercises 1–6, use the diagram above.

1. a. List five other pairs of vertical angles in the diagram.

b. List five other pairs of corresponding angles in the diagram.

2. What other segments form equal ratios? Explain.

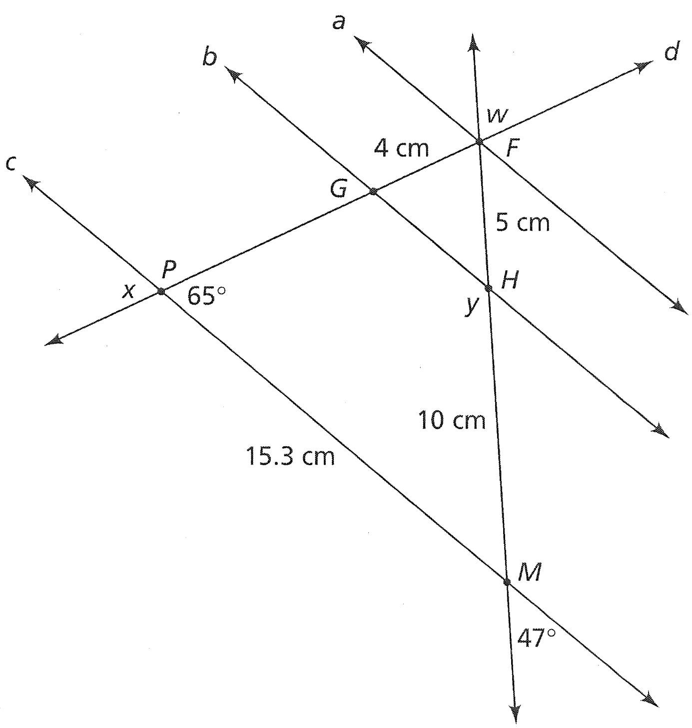
3. What angles are congruent to $\angle CEK$? Explain.

4. What angles are congruent to $\angle BDJ$? Explain.

5. Why are $\angle EDL$ and $\angle BDJ$ congruent?

6. What other triangle is similar to triangle ABC? Explain.

For Exercises 7–13, use the diagram below. Lines *a*, *b*, and *c* are parallel.



7. What are the measures of $\angle PMH$ and $\angle x$?

8. What are the measures of $\angle FGH$ and $\angle GHF$?

9. What is the measure of $\angle PGH$?

10. What is the measure of $\angle y$?

11. What is the length of $\overline{GP}$?

12. What is the measure of $\angle GFH$ and $\angle w$?

13. What is the length of $\overline{GH}$?

Remember that lines are parallel if their slopes are equal and lines are perpendicular if their slopes are negative reciprocals of each other.

Sample Are the lines $y = 2x + 8$ and $3 = 2x - y$ parallel?

Rewrite the second equation.

$$3 = 2x - y$$
$$y + 3 = 2x - y + y$$
$$y + 3 = 2x$$
$$y = 2x - 3$$

The slope of this line is 2, which is also the slope of the first line. The slopes are equal, so the lines are parallel.

Sample Are the lines $y = 2x + 8$ and $7 = \frac{1}{3}x - y$ perpendicular?

Rewrite the second equation.

$$7 = \frac{1}{3}x - y$$
$$y + 7 = \frac{1}{3}x - y + y$$
$$y + 7 = \frac{1}{3}x$$
$$y = \frac{1}{3}x - 7$$

The slope of this line is $\frac{1}{3}$. The slope of the first line is 2. The slopes are not negative reciprocals of each other, so the lines are not perpendicular.

Determine whether each pair of lines is parallel, perpendicular, or neither.

14. $y = 5x - 7$

$y + 5x = 12$

15. $y = x - 0.5$

$y + x = 0.25$

16. $y = \frac{1}{2}x - \frac{3}{4}$

$y - \frac{1}{2}x = \frac{5}{6}$

17. $2y = 6x - 72$

$y - 3x = 15$

18. $y + x = 12$

$y - x = 12$

19. $5x - y = 12$

$5y + x = 35$

At a Glance

Topic 6: Parallel and Perpendicular

PACING 1 day

Mathematical Goals

- Investigate parallel and perpendicular algebraically and geometrically
- Apply properties of angle pairs formed by parallel lines and transversals
- Understand properties of the ratio of segments when parallel lines are cut by transversals

Guided Instruction

Materials
- Labsheet 6.1

Vocabulary
- transversals
- corresponding angles
- vertical angles

The terminology of corresponding angles and vertical angles may be new to students. Corresponding angles are congruent only if they are formed by a transversal intersecting parallel lines. If they are formed by a transversal intersecting non-parallel lines, then the corresponding angles are *not* cougruent. Vertical angles are always congruent. Students use these facts to identify pairs of congruent angles. Before students begin to solve the exercises, you may want to help students remember what they know about proportional reasoning and similar triangles.

Because angles are named with triads of letters, many of the angles in the figure on the first page have multiple names. The letter representing the vertex of the angle will be the same for each of the names. For example, $\angle GAC$ can also be named $\angle GAE$ and $\angle GAM$. The answers given below use only one name for each angle. Students may use different names for the angles.

Students may have some difficulty identifying five pairs of vertical and corresponding angles in Exercise 1. You may want to ask students to work with a partner or in small groups to find all of these pairs of angles. One strategy that may help students find corresponding angles is to imagine that one of the transversals is removed from the figure. Students might actually cover up one of the transversals with a finger.

For the topic introduction, ask:

- *What does perpendicular mean?*
- *What does parallel mean?*
- *Can you name angles* ACI *or* CEK *in more than one way?*
- *What does it mean for ratios to be equal?*
- *Suppose that one of the transversals is removed. Is it easier to find pairs of corresponding angles?*

For the section after Exercise 13, ask:

- *What are negative reciprocals?*
- *What is the slope of a line?*

Assignment Guide for Topic 6

Core 1–19

Answers to Topic 6

Exercises

1. **a.** Any five of the following will do: $\angle XAY$ and $\angle BAC$, $\angle XAF$ and $\angle GAC$, $\angle YAG$ and $\angle FAB$, $\angle ACB$ and $\angle ICE$, $\angle ACI$ and $\angle BCE$, $\angle ABC$ and $\angle HBD$, $\angle ABH$ and $\angle CBD$, $\angle CEK$ and $\angle DEM$, $\angle KEM$ and $\angle CED$, $\angle BDE$ and $\angle JDL$, $\angle BDJ$ and $\angle LDE$

   b. Any five of the following will do:
   $\angle XAG$ and $\angle ACI$, $\angle XAG$ and $\angle CEK$, $\angle ACI$ and $\angle CEK$

   $\angle YXA$ and $\angle GAC$, $\angle YXA$ and $\angle ICE$, $\angle YXA$ and $\angle KEM$, $\angle GAC$ and $\angle ICE$, $\angle GAC$ and $\angle KEM$, $\angle ICE$ and $\angle KEM$

   $\angle XAF$ and $\angle ACB$, $\angle XAF$ and $\angle CED$, $\angle ACB$ and $\angle CED$

   $\angle FAC$ and $\angle BCE$, $\angle FAC$ and $\angle DEM$, $\angle BCE$ and $\angle DEM$

   $\angle YAG$ and $\angle ABC$, $\angle YAG$ and $\angle BDE$, $\angle ABC$ and $\angle BDE$

   $\angle GAB$ and $\angle CBD$, $\angle GAB$ and $\angle EDL$, $\angle CBD$ and $\angle GAB$

   $\angle XYA$ and $\angle FAB$, $\angle XYA$ and $\angle HBD$, $\angle XYA$ and $\angle JDL$, $\angle FAB$ and $\angle HBD$, $\angle FAB$ and $\angle JDL$, $\angle HBD$ and $\angle JDL$

   $\angle YAF$ and $\angle ABH$, $\angle YAF$ and $\angle BDJ$, $\angle ABH$ and $\angle BDJ$

2. Proportions can be written in multiple ways, so the answers below are samples.
   $\overline{AX} : \overline{XY} = \overline{AC} : \overline{CB} = \overline{AE} : \overline{ED}$
   $\overline{AY} : \overline{XY} = \overline{AB} : \overline{BC} = \overline{AD} : \overline{DE}$
   $\overline{AX} : \overline{AY} = \overline{AC} : \overline{AB} = \overline{AE} : \overline{AD}$
   Corresponding parts of similar triangles are congruent. The transversals create three similar triangles: triangle AXY, triangle ACB, and triangle AED.

3. $\angle CEK$ is congruent to $\angle ACI$ and $\angle XAG$ (corresponding angles), and $\angle DEM$, $\angle BCE$, and $\angle FAC$ (vertical angles for the first three angles).

4. $\angle BDJ$ is congruent to $\angle ABH$ and $\angle YAF$ (corresponding angles), and $\angle EDL$, $\angle CBD$, and $\angle GAB$ (vertical angles for the first three angles).

5. They are vertical angles.

6. Triangle AED is similar, because the angles are congruent. Triangle AYX is similar, because the angles are congruent.

7. $\angle PMH$ is 47°, $\angle x$ is 65°.

8. $\angle FGH$ is 65°, $\angle GHF$ is 47°.

9. $\angle PGH$ is 115°.

10. $\angle y$ is 133°.

11. $\overline{GP}$ is 8 cm. **Note:** $\triangle FGH \sim \triangle FPM$ and the scale factor is 3.

12. $\angle GFH$ is 68°, $\angle w$ is 68°.

13. 5.1 cm

14. Neither; the slopes are $+5$ and -5.

15. Perpendicular; the slopes are $+1$ and -1 (negative reciprocals).

16. Parallel; the slopes are $\frac{1}{2}$ and $\frac{1}{2}$ (equal).

17. Parallel; the slopes are 3 and 3 (equal).

18. Perpendicular; the slopes are 1 and -1 (negative reciprocals).

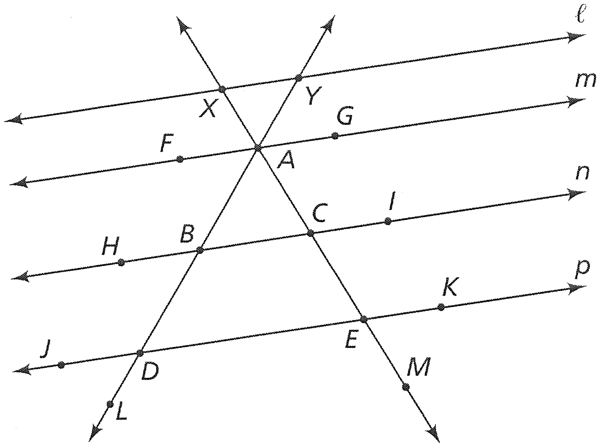
19. Perpendicular; the slopes are 5 and $-\frac{1}{5}$ (negative reciprocals).

Name ______________________ Date __________ Class __________

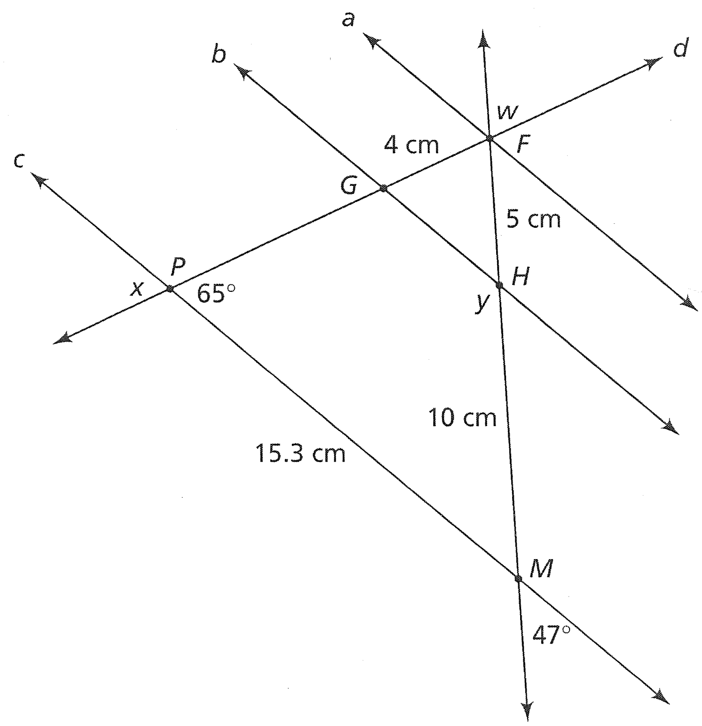
Labsheet 6.1

Topic 3

Introduction



Exercises 8–14



Investigation 3

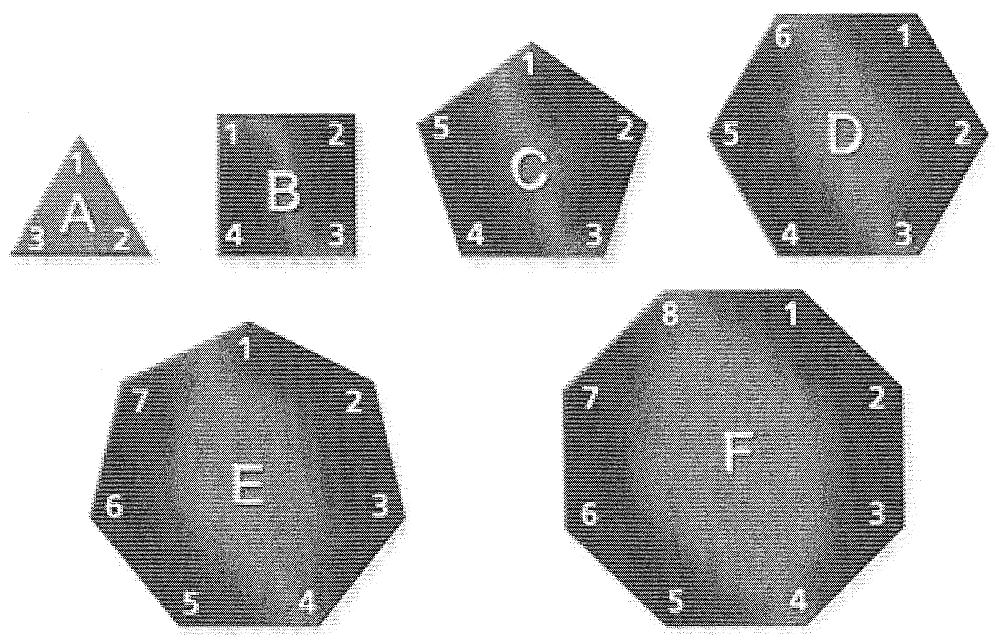
Polygon Properties and Tiling

You learned about angles and angle measure in Investigations 1 and 2. What you learned can help you figure out some useful properties of the angles of a polygon. Let's start with the sum of the measures of all the inside angles at the vertices of a polygon. This sum is called the **angle sum** of a polygon.

3.1 Angle Sums of Regular Polygons

TEKS / TAKS 6(6)B Identify relationships involving angles in triangles and quadrilaterals. 6(8)C Measure angles. 6(13)A Make conjectures.

Below are six regular polygons that are already familiar to you.



What is the angle sum of each figure?

Do you see a pattern relating the number of sides to the angle sum?

Problem 3.1 Angle Sums of Regular Polygons

A. 1. In Problem 2.3, you measured the angles of some regular polygons—triangles, squares, and hexagons. Record the number of sides, the angle measures, and the angle sum of a triangle, square, and hexagon in a table like the one below.

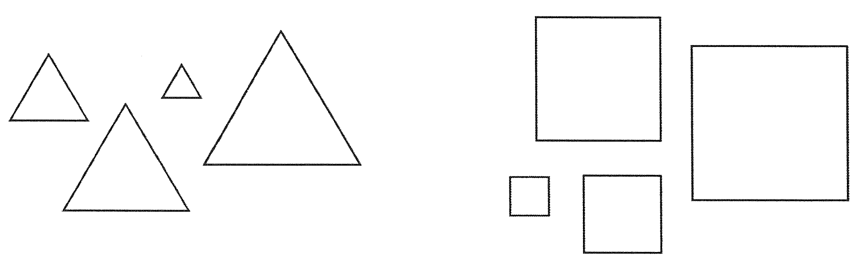
active math online
For: Angle Sum Activity
Visit: PHSchool.com
Web Code: amd-3301

Polygon	Number of Sides	Measure of an Angle	Angle Sum
Triangle			
Square			
Pentagon			
Hexagon			
Heptagon			
Octagon			
Nonagon			
Decagon			

2. Measure an angle of the regular pentagon and regular octagon from your Shapes Set. Record the measures of the angles and the angle sums in your table. What patterns do you see?

3. Use your patterns to fill in the table for a regular polygon with seven, nine, and ten sides.

B. Below are two sets of regular polygons of different sizes. Do the same patterns relating the number of sides, the measures of the angles, and the angle sums apply to these shapes? Explain.



C. Describe how you could find the angle sum of a regular polygon that has N sides.

ACE **Homework starts on page 62.**

3.1 Angle Sums of Regular Polygons

Goals

- Find angle sums of regular polygons
- Determine relationships between the number of sides and the angle sum of a regular polygon

Launch 3.1

Remind students what a regular polygon is.

- *In Investigation 1, you experimented with the six regular polygons shown in your books. Someone remind us what a regular polygon is.* (A regular polygon is one in which all sides are the same size and all angles are the same size.)

Take a few suggestions, and clarify them until their definition is correct.

- *In Investigation 1, you were trying to find out which of these shapes would fit together nicely like tiles on a floor. You want to continue to think about these regular polygons by investigating the size of their angles and what happens to the measures of the angles as the number of sides increases. The sum of the angles of a polygon is called the "angle sum."*

Students should naturally understand that the angles being discussed are the angles "inside" the polygon, or interior angles. Interior angle is a term that is not necessary at this point and will formally be introduced in Problem 3.4.

Suggested Questions Put up Transparency 3.1A or refer to the textbook.

- *Which polygon has angles that appear to be the smallest?*
- *Which polygon has angles that appear to be the largest?*

These questions are asking students to make informal observations about the sizes of the interior angles without using measuring tools. They should be able to answer these questions from their exploration in Problem 2.3. At this point, students should be able to see that the size of the interior angles increases as the number of sides increases. One way to help struggling students see this is to demonstrate at the overhead projector how the sizes of the angles compare by placing one shape on top of another.

Arrange the students in groups of 2 and 3.

Explore 3.1

During this time, students can begin by entering the data they collected in Problem 2.3. They should continue measuring and recording measurements of a regular pentagon and octagon.

For students who see the patterns quickly, ask them to make a general rule. Some may even be ready to use symbols. Have the students use their rule to find the angle sums for a polygon with seven, nine, and ten sides.

Summarize 3.1

Display Transparency 3.1A. With input from the class, fill in the missing information. Accept, record, and then discuss all answers.

For example, many students will have angle measures that are close to the actual measures but not exact. Some students may disagree with the measurements others give.

Classroom Dialogue Model

One teacher handled the disagreement on angle measures in the following manner:

Teacher *I have listed the names of the five regular polygons you had to measure. For the triangle, what numbers do you have to fill in the next three columns? Does anyone have anything different?*

When a group answered yes, the teacher also recorded its numbers in the appropriate columns. For example, one group said the pentagon has five sides and each angle measures 100°, for a total of 500°. Another group said that the pentagon has five sides and each angle measures 108° for a total of 540°. The teacher listed both of these groups' answers and continued to collect information on the remaining polygons. When the chart was complete, she started to question the data.

Teacher *Why do we have different answers when we all measured the same angles? Does anyone have a suggestion for how we might resolve the angle measures we disagree on?*

In this class, many students suggested measuring the interior angles again. This resulted in some of the measures being eliminated from the chart. However, some students had given smaller angle measurements for octagons than hexagons.

Teacher *Look at all the answers that are now recorded on the chart. Are there any that don't seem reasonable?*

Students argued for elimination of some of the measures. The teacher only removed numbers from the chart when students had given a mathematical reason for eliminating them.

Teacher *What patterns do you notice in the way the size of the angles is increasing? What patterns do you notice in the way the size of the angle sum is increasing?*

Some students noticed that the angle sum seemed to be increasing by about 180° with each additional side. As a result, more numbers were eliminated from the chart because they did not fit the pattern. The teacher continued with the discussion until the class had arrived at the correct measurements. See Figure 1 below.

The teacher then tried to extend their thinking by asking *if-then* questions:

Teacher *If a regular polygon has twenty sides, what will be the sum of all the angles in that polygon? Explain why your answer makes sense.*
If a regular polygon has twenty sides, each angle must have how many degrees? Explain.

If you gently encourage students to make observations about patterns in the chart, some may look at the angle sums and observe the relationship to the triangle's 180° angle sum. You may have a student who extends this relationship by noticing that a square contains two triangles (by drawing one diagonal, $180° \cdot 2 = 360°$), a pentagon contains three triangles (by drawing two diagonals from one vertex, $180° \cdot 3 = 540°$), and so on. This will be useful for the Launch to Problem 3.2. If students are not ready, it's not necessary to force the issue now.

Question B tries to expand the ideas students have just developed. Students are asked to consider what happens to the angles of regular polygons when the lengths of the sides change. They measure the angles of the set of regular triangles and regular quadrilaterals and observe that they remain the same regardless of side length. The larger figures are *similar* to the smaller figures. In similar figures, angles are the same. These ideas are developed more fully in *Stretching and Shrinking.*

This is a good time to reinforce that changing a side length does not affect the size of an angle. An equilateral triangle is the easiest to use. Put three different ones on the overhead and ask about their angle measures and side lengths. Repeat this for one or two other polygons. Point out that two polygons with the same number of sides and equal corresponding angles are not necessarily *congruent* (same shape, same size).

Figure 1

Polygon	Number of Sides	Measure of an Angle	Angle Sum
Triangle	3	60°	180°
Square	4	90°	360°
Pentagon	5	108°	540°
Hexagon	6	120°	720°
Heptagon	7	128.6°	900°
Octagon	8	135°	1,080°
Nonagon	9	140°	1,260°
Decagon	10	144°	1,440°

3.1 Angle Sums of Regular Polygons

At a Glance

PACING 1 day

Mathematical Goals

- Find angle sums of regular polygons
- Determine relationships between the number of sides and the angle sum of a regular polygon

Launch

Remind students what a regular polygon is.

- *We call the sum of the interior angles of a polygon the "angle sum."*

Arrange six regular polygons on the overhead or refer to the textbook.

- *Which polygon has angles that appear to be the smallest?*
- *Which polygon has angles that appear to be the largest?*

Make sure that students see that the size of the interior angles increases as the number of sides increases. Demonstrate at the overhead projector how the sizes of the angles compare by placing one shape on top of another. Arrange students in groups of 2 and 3.

Materials

- Transparency 3.1A and 3.1B
- Overhead Shapes Set (Transparency 1.1E)
- Blank transparencies
- Overhead markers

Vocablulary

- angle sum

Explore

During this time, students begin by organizing the data they collected in Problem 2.3. They should continue measuring and recording measurements of a regular pentagon and octagon.

For students who see the patterns quickly, ask them to make a general rule. Some may even be ready to use symbols. Have the students use their rule to find the angle sums for a polygon with seven, nine, and ten sides.

Materials

- Student data from Problem 2.3
- Angle rulers
- Shapes Sets (1 per group)

Summarize

Display Transparency 3.1A. Accept and record all student answers on the chart. Some students may disagree with the measurements others give. When the chart is complete, you may want to question the data.

- *Why do we have different answers when we all measured the same angles? How might we resolve the angle measures we disagree on?*
- *Look at all the answers that are now recorded on the chart. Are there any that don't seem reasonable?*

Remove numbers from the chart when students have given a mathematical reason for eliminating them.

- *What patterns do you notice in the way the size of the angles is increasing? What patterns do you notice in the way the size of the angle sum is increasing?*

Continue with the discussion until the class arrives at the correct measurements. See the extended Summarize for more details and questions.

Materials

- Student notebooks

continued on next page

Summarize

continued

Put three different-sized equilateral triangles on the overhead and ask about their angle measures and then about the side lengths. Repeat this for one or two other polygons. Point out that two polygons with the same number of sides and equal corresponding angles are not necessarily identical.

ACE Assignment Guide for Problem 3.1

Differentiated Instruction
Solutions for All Learners

Core 1–2

Other *Connections* 15–16 (angle ruler needed for 15), *Extensions* 21

Adapted For suggestions about adapting Exercise 2 and other ACE exercises, see the CMP *Special Needs Handbook.*

Answers to Problem 3.1

A. 1. and 3. After part (3), the completed table should look like Figure 2.

2. Students should notice that the measure of the interior angles increases with the number of sides. They should also notice that, starting from the triangle, the angle sum increases by 180° with each additional side. The angle measure increases by less as the number of sides goes up.

B. Students should conclude that the length of the sides in the sets of similar polygons has no effect on the angle sum or the measure of the interior angles.

C. If the number of sides is N, then the angle sum of the polygon is $N - 2$ times 180°. Or: Angle Sum $= 180°(N - 2)$.

Figure 2

Polygon	Number of Sides	Measure of an Angle	Angle Sum
Triangle	3	60°	180°
Square	4	90°	360°
Pentagon	5	108°	540°
Hexagon	6	120°	720°
Heptagon	7	128.6°	900°
Octagon	8	135°	1,080°
Nonagon	9	140°	1,260°
Decagon	10	144°	1,440°

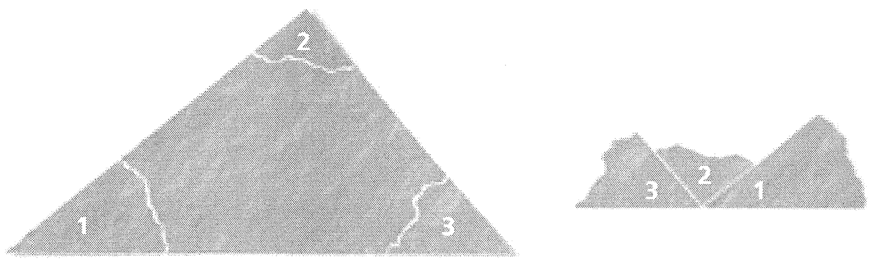
3.2 Angle Sums of Any Polygon

TEKS / TAKS 6(6)B Identify relationships involving angles in triangles and quadrilaterals.
6(13)A Make conjectures from patterns.

Do the patterns that you observed for the angle sum of regular polygons apply to all polygons?

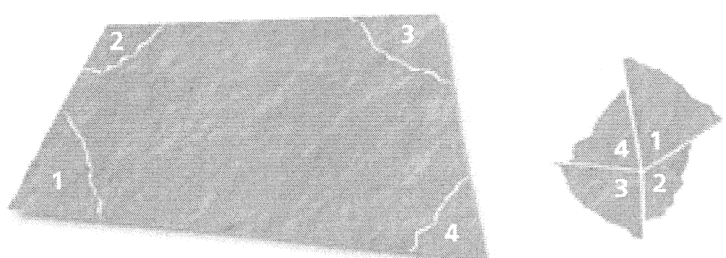
Getting Ready for Problem 3.2

Suppose you tear the three corners off of a triangle. You can arrange them this way:



- Based on the picture, what is the sum of angles 1, 2, and 3? How do you know?
- Make a conjecture about the angle sum of any triangle.

You could do the same thing with a quadrilateral.

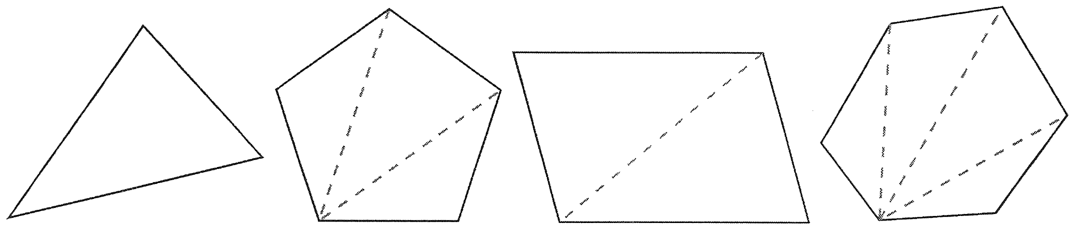


- Based on the picture, what is the sum of angles 1, 2, 3, and 4? How do you know?
- Make a conjecture about the angle sum of any quadrilateral.
- Do similar patterns hold for other polygons?

Problem 3.2 Angle Sums of Any Polygon

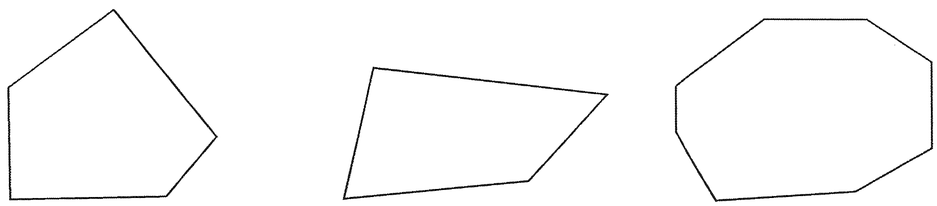
Tia and Cody claim that the angle sum of any polygon is the same as the angle sum of a regular polygon with the same number of sides. They use diagrams to illustrate their reasoning.

A. Tia divides polygons into triangles by drawing all the *diagonals* of the polygons from one vertex, as in the diagrams below:



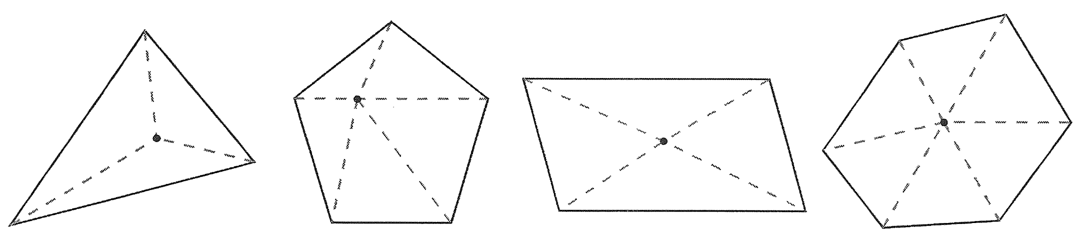
1. Study Tia's drawings. How can you use Tia's method to find the angle sum of each polygon?

2. Copy these three polygons. Use Tia's method to find the angle sum of each polygon.



3. Does Tia's method work for any polygon? Explain.

B. Cody also discovered a method for finding the angle sum of any polygon. He starts by drawing line segments from a point within the polygon to each vertex.



1. Study Cody's drawings. How can you use Cody's method to find the angle sum of each polygon?

2. Copy the three polygons from Question A part (2). Use Cody's method to find the angle sum of each polygon.

3. Does Cody's method work for any polygon? Explain.

C. In Problem 3.1, you found a pattern relating the number of sides of a regular polygon to the angle sum. Does the same pattern hold for any polygon? Explain.

ACE **Homework starts on page 62.**

Name ______________________ Date ____________ Class ____________

Labsheet 3.2

Angle Sums

Polygon	Number of Sides	Angle Sum Using Tia's Method	Angle Sum Using Cody's Method	Angle Sum
Triangle	3			
Quadrilateral	4			
Pentagon	5			
Hexagon	6			
Heptagon	7			
Octagon	8			
Nonagon	9			
Decagon	10			

3.2 Angle Sums of Any Polygon

Goals

- Develop informal arguments for conjectures about the relationship between the number of sides and the angle sum of any polygon
- Find angle sums of any polygon

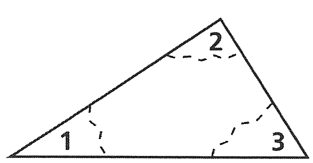
Students use their knowledge of 180° and 360° to find the sum of the angles of a polygon. In the Getting Ready for Problem 3.2, we start with a triangle and determine that any triangle has an angle sum of 180°. We then look at a quadrilateral and determine that any quadrilateral has an angle sum of 360°. Then they use what they know about the angle sum of a triangle to find the angle sum of other polygons by subdividing the polygon into non-overlapping triangles.

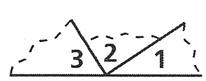
Launch 3.2

Suggested Questions Begin by asking students whether or not they think the angle sum formula or pattern for regular polygons they developed in Problem 3.1 will hold for polygons in general.

- *Do you think the angle sum of any triangle is 180°? How can you check?*

Draw a triangle on a sheet of paper or a transparency and label each angle 1, 2, and 3. After cutting out the triangle, tear (or cut) off all three angles and arrange the angles around a point on another sheet of paper or on the overhead.



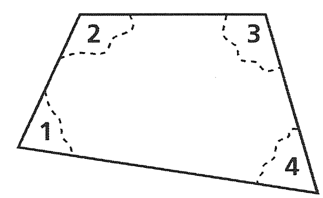


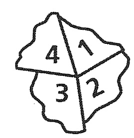
- *What do you observe about the sum of the angles of the triangle?* (Since the three angles form a straight line, the sum of the angles is 180°.)

Repeat the experiment with different shaped triangles.

- *Based on your experiments, what is the angle sum of any triangle?* (180°)
- *What if this experiment were repeated for a quadrilateral? Can you predict the sum of the angles of the quadrilateral?*

Draw a quadrilateral on a sheet of paper or a transparency and label each angle 1, 2, 3, and 4. After cutting out the quadrilateral, tear (or cut) off all four angles and arrange the angles around a point on another sheet of paper or on the overhead.





- *Based on the picture, what is the sum of angles 1, 2, 3, and 4? How do you know?* (360°, because the angles fit around a point.)
- *Make a conjecture about the angle sum of any quadrilateral.* (The angle sum of any quadrilateral is 360°.)
- *Do similar patterns hold for other polygons?* (Students may predict that the angle sums match the results they got in Problem 3.1.)

Some students may need to be reminded that the sum of the angles around a point is 360°.

- Now describe both Tia's and Cody's method for finding the sum of the angles of a polygon with sides greater than three. You could have half the students analyze one method and the others analyze the other method. In summary, a person from each group would present the argument for the reasoning in the method they explored. You could also assign one for the class to work on in class and then assign the other for homework.

Arrange the students into groups of 2 and 3.

Explore 3.2

Students should begin exploring Tia's method. If they are having trouble understanding Tia's drawings, remind them that the angle sum of every triangle is 180°.

For students having a hard time seeing that the sum of the angles of a polygon is equal to the sum of the angles in the $(N - 2)$ triangles, you can suggest numbering the angles of the triangles in Tia's method.

A similar numbering method may also help with Cody's method. Some students may need to be reminded that the sum of the angles around a point is 360°.

For students who see the patterns quickly, ask them to make a general rule. Ask how their answers compare to their answers to Problem 3.1.

Summarize 3.2

Have someone explain each method. Be sure the class understands the explanations. Let them ask questions.

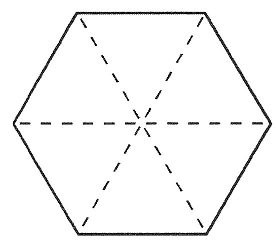
Suggested Questions

- *Are the angle sums of these polygons the same as the angle sums for regular polygons with the same number of sides?* (Yes.)
- *What about the measure of each interior angle?*
  (No, because in a regular polygon each interior angle has the same size, and in these polygons, the angles are not necessarily the same size.)

Suggested Questions Ask the class:

- *What is the angle sum of a 12-sided polygon? A 100-sided polygon?*
  (A 12-sided polygon has an angle sum of 1,800° and a 100-sided polygon has an angle sum of 17,640°.)
- *What is the angle sum of any polygon with N sides?* $(180° \times (N - 2))$
- *Use the rule to find the angle sum of a polygons with 50 sides.* (8,640°)

Here is an example:



For each shape you can use either method. For the hexagon using Cody's method:
$180° \times 6 - 360° = 1{,}080° - 360° = 720°$.

For the pentagon using Tia's method:
$(N - 2) \times 180° = (5 - 2) \times 180° = 540°$, where N is the number of sides.

You can modify the chart in Problem 3.1 so that students see that the angle sum of any polyhedron is $180° \times (N - 2)$. Use this chart to launch the next problem.

3.2 Angle Sums of Any Polygon

At a Glance

PACING 1 day

Mathematical Goals

- Develop informal arguments for conjectures about the relationship between the number of sides and the angle sum of any polygon
- Find angle sums of any polygon

Launch

Ask students whether or not they think the angle sum formula or pattern for regular polygons will hold for polygons in general.

- *Do you think the angle sum of any triangle is 180°? How can we check?*

Draw a triangle on a sheet of paper or a transparency and label each angle 1, 2, and 3. After cutting out the triangle, tear (or cut) off all three angles and arrange the angles around a point on another sheet of paper or on the overhead.

- *What do you observe about the sum of the angles of the triangle?*

Repeat the experiment with different shaped triangles.

- *Based on your experiments, what is the angle sum of any triangle?*
- *What if this experiment was repeated for a quadrilateral?*
- *Can you predict the sum of the angles of the quadrilateral?*

Draw a quadrilateral on a sheet of paper or a transparency and label each angle 1, 2, 3, and 4. After cutting out the quadrilateral, tear (or cut) off all four angles and arrange the angles around a point or on the overhead.

- *Based on the picture, what is the sum of angles 1, 2, 3, and 4? How do you know?*
- *Make a conjecture about the angle sum of any quadrilateral.*
- *Can you predict the sum of the angles of a pentagon? A hexagon? Any* N*-sided polygon?*

Describe Cody and Tia's methods for finding the angle sum of a polygon. Arrange the students into groups of 2 and 3.

Materials

- Transparency 3.2
- Overhead Shapes Set (Transparency 1.1E)
- Construction paper and scissors

Explore

Students should begin exploring Tia's method. You may need to remind students that the angle sum of every triangle is 180°, and that the sum of the angles around a point is 360°. For students having a hard time seeing that the sum of the angles of a polygon are equal to the sum of the angles in the $(N - 2)$ triangles, you can suggest numbering the angles of the triangles in Tia's method. For students who see the patterns quickly, ask them to make a general rule. Ask how their answers compare to their answers to Problem 3.1.

Materials

- Labsheet 3.2
- Shapes Sets (1 per group)

continued on next page

Summarize

continued

Have someone explain each method.

- *Are the angle sums of these polygons the same as the angle sums for regular polygons with the same number of sides?*
- *What about the measure of each interior angle?*

Ask the class:

- *What is the angle sum of a 12-sided polygon? A 100-sided polygon? What is the angle sum of any polygon with* N *sides?*

Materials

- Student notebooks

ACE Assignment Guide for Problem 3.2

Differentiated Instruction
Solutions for All Learners

Core 3–6, 9

Other *Applications* 7, 8, 10; *Connections* 17; *Extensions* 22–24; unassigned choices from previous problems

Adapted For suggestions about adapting ACE exercises, see the CMP *Special Needs Handbook*.

Answers to Problem 3.2

A. 1. By drawing diagonals from a vertex of the polygon, Tia sectioned off triangles inside the polygon. The number of triangles created inside the polygon is always two less than the number of sides of the polygon. Since the sum of the angles of a triangle equals 180°, the sum of the angles of a polygon is the number of triangles drawn inside times 180°, or (two less than the number of sides) times 180°.

2. The angle sum of the pentagon is 540°, the angle sum of the quadrilateral is 360°, and the angle sum of the octagon is 1,080°.

3. Yes. When you draw diagonals from a vertex of a polygon you get triangles, whose angles sum to 180° times the number of triangles. If you use this mathematical statement:
(Number of sides of polygon − 2) × 180°,
you will get the angle sum of the polygon. This is consistent with what we learned about regular polygons.

B. 1. Cody drew line segments from a point inside of each polygon to all of its vertices, thereby forming triangles inside the polygon. He then discovered that if you multiplied the number of triangles by 180° (finding the total angle measurements of all the triangles created), and subtracted 360° from this sum (for the angles around the interior point that are not part of the polygon's angles), the result is the sum of the interior angles of the original polygon.

2. The angle sum of the pentagon is 540°, the angle sum of the quadrilateral is 360°, and the angle sum of the octagon is 1,080°.

3. Yes. We could use this mathematical statement:
(Number of triangles) × 180° − 360° to get the angle sum of the polygons.

C. Yes. The angle sum of a regular polygon with N sides is the same as the angle sum of any polygon with N sides.

Name ______________________________ Date __________ Class __________

Additional Practice

1. An isosceles triangle has two 50° angles. What is the measure of the third angle? Explain how you found your answer.

2. One angle of an isosceles triangle measures 100°. What are the measures of the other two angles? Explain your reasoning.

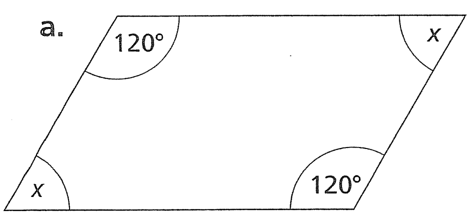
3. Two of the angles of a parallelogram each measure 75°. What are the measures of the other two angles? Explain your reasoning.

4. One angle of a parallelogram measures 40° and another angle measures 140°. What are the measures of the other two angles? Explain how you found your answer.

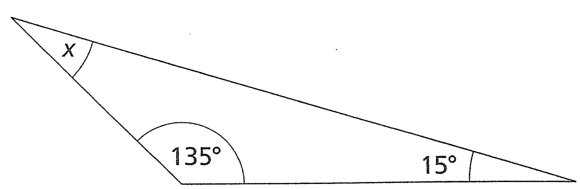
5. Can a parallelogram have two 45° angles and two 75° angles? Why or why not?

6. Fc each of the shapes below, find the unknown angle measure without using your angle ruler.

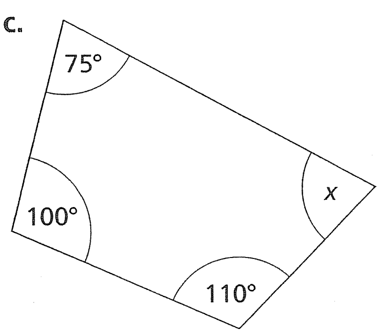
a.



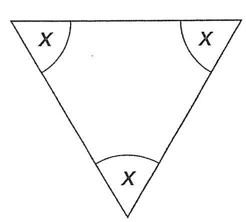
b.



c.



d.



Name ____________________ Date __________ Class __________

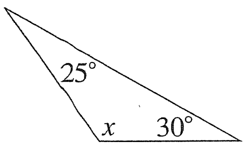
Skill: Angle Sums and Exterior Angles of Polygons

Investigation 3

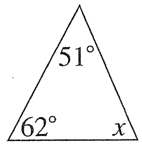
Shapes and Designs

Find the measure of each angle labeled *x*.

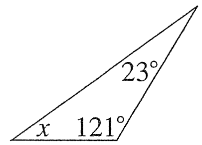
1.



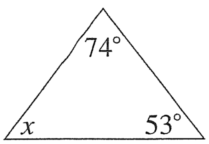
2.



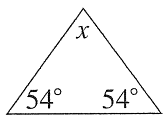
3.



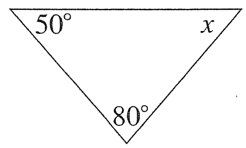
4.



5.

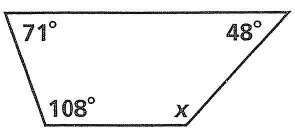


6.

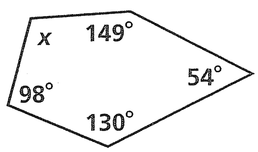


Find the measure of each angle labeled *x*.

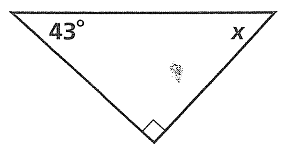
7.



8.

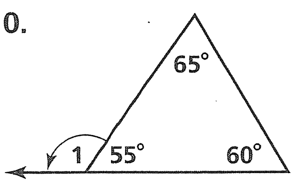


9.

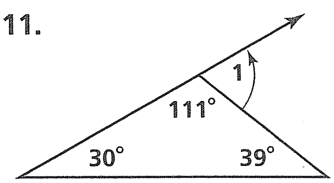


Find the measure of angle 1 in each figure.

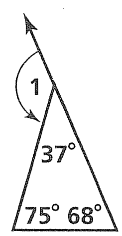
10.



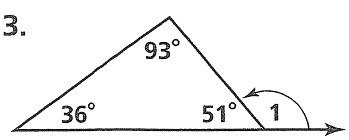
11.



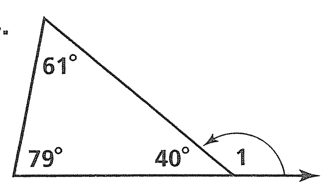
12.



13.



14.



15.